Instructor: Patricia Wrean Name: Solution Set

MATH 156
Practice Final

Total = $\frac{\quad}{80}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app.

GOOD LUCK!

1. (10 points) Convert the following numbers into the indicated base. Give exact answers (do not round) and show your work.

(a) 702.16_8 to hexadecimal

$1C2.38_{16}$

$702.16_8 = 111\ 000\ 010.\ 001\ 110_2$

$= 1\ 1100\ 0010.\ 0011\ 1000_2$

$= 1\ C\ 2\ .\ 3\ 8_{16}$

3

(b) 5152_6 to decimal

1148 (or 1148_{10})

$5152_6 = 5\times6^3 + 1\times6^2 + 5\times6^1 + 2\times6^0$

$= 1080 + 36 + 30 + 2$ ← can skip this step

$= 1148$

3

(c) 0.8 to binary

$0.\overline{1100}_2$

$0.8 \times 2 = 1 + 0.6$

$0.6 \times 2 = 1 + 0.2$

$0.2 \times 6 = 0 + 0.4$

$0.4 \times 2 = 0 + 0.8$

4

① correct method
① first two digits
① next two digits
① correct repeater bar

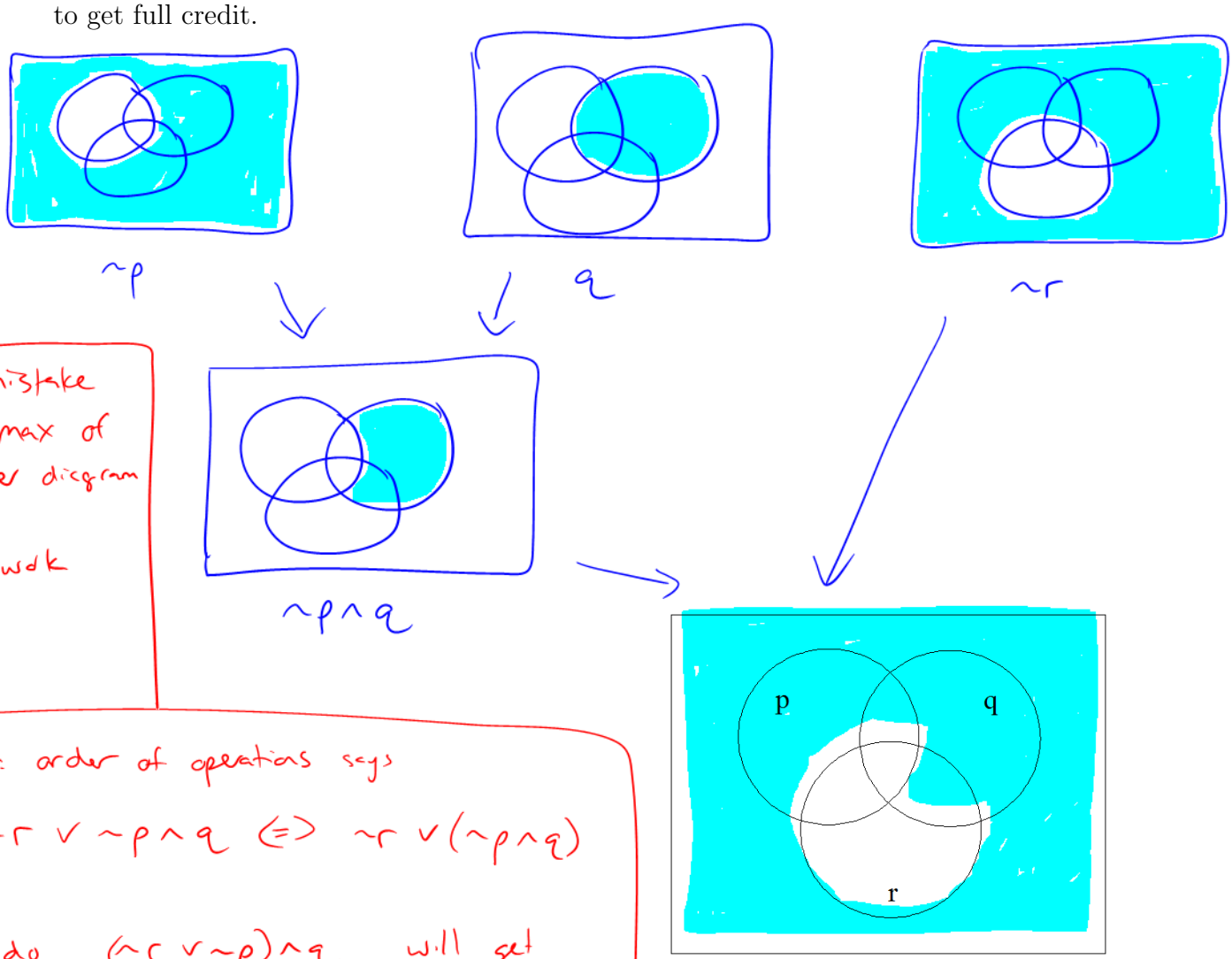
2. (3 points) For each pair of sentences below, is the second sentence the negation of the first? Answer by circling the correct choice.

   (a) There are at least 4 bugs in the program. There are at most 4 bugs in the program. Yes / No

   (b) There are no batteries left. There is at least one battery left. Yes / No

   (c) All of the berries are ripe. None of the berries are ripe. Yes / No

3. (4 points) Represent $\sim r \vee \sim p \wedge q$ on the following Venn diagram by shading in the appropriate regions. Show intermediate steps on separate sketches and label them clearly to get full credit.



-1 per mistake for max of -2 per diagram

-3 no work

note: order of operations says

$\sim r \vee \sim p \wedge q \iff \sim r \vee (\sim p \wedge q)$

if do $(\sim r \vee \sim p) \wedge q$, will get

-1

4. (4 points) Is the expression $\sim q \oplus (p \wedge \sim q)$ logically equivalent to $\sim(p \vee q)$? Use a truth table to justify your answer.

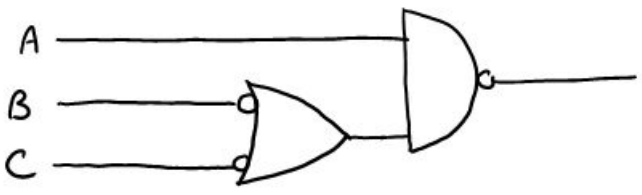
p	q	$\sim q$	$p \wedge \sim q$	$\sim q \oplus (p \wedge \sim q)$	$p \vee q$	$\sim(p \vee q)$
0	0	1	0	1	0	1
0	1	0	0	0	1	0
1	0	1	1	0	1	0
1	1	0	0	0	1	0

yes

$-\frac{1}{2}$ each mistake for max of -1 per column

-1 no conclusion

5. (2 points) Write the Boolean expression that corresponds to the following gate diagram. Do not simplify!



$\overline{A(\bar{B} + \bar{C})}$

or $\overline{(\bar{B} + \bar{C})A}$

-1 if forgot brackets

-1 if mixed up and vs or

6. (4 points) Simplify the following using the laws of logic: use one law of logic per line, and be sure ~~to state~~ the name of the law you are ~~using~~. If you're stuck, try using a truth table for part marks. if used truth tbl, (-2)

$$\overline{\overline{B}+C}(\overline{B}+\overline{C})+B$$

method 1:

$\overline{\overline{B}+C}(\overline{B}+\overline{C})+B$

$B\overline{C}(\overline{B}+\overline{C})+B$ DeMorgans

$B\overline{C}\,\overline{B}+B\overline{C}\overline{C}+B$ distributive

$\overline{C}\cdot 0+B\overline{C}\overline{C}+B$ complement

$0+B\overline{C}\overline{C}+B$ identity

$B\overline{C}\overline{C}+B$ identity

$B\overline{C}+B$ idempotent

B absorption

method 2:

$\overline{\overline{B}+C}(\overline{B}+\overline{C})+B$

$B\overline{C}(\overline{B}+\overline{C})+B$ DeMorgans

B absorption

(-1) each mistake

(-½) name of law missing or incorrect

7. (4 points) The following statement is true: "If and only if you forget your password, then you will not be able to log on." Given that, answer the following questions by indicating the correct choice.

(a) You forgot your password. Will you be able to log on? Yes / No / Maybe

(b) You were able to log on. Did you forget your password? Yes / No / Maybe

(c) You were not able to log on. Did you forget your password? Yes / No / Maybe

(d) You did not forget your password. Will you be able to log on? Yes / No / Maybe

(1) each

no maybes

"if and only iff", sc biconditional

either both true or both false

8. (4 points) Consider the statement $p \to q$: "If you wear a bowtie, then you will be fashionable." Which of the following statements are logically equivalent to $p \to q$? Indicate all of the correct answers.

(1) each

(a) If you are fashionable, then you wore a bowtie. $q \to p$ converse

(b) If you are not fashionable, then you did not wear a bowtie. $\sim q \to \sim p$ contrapositive

(c) If you don't wear a bowtie, you won't be fashionable. $\sim p \to \sim q$ inverse

(d) Either you did not wear a bowtie or you are fashionable or both. $\sim p \vee q$ "or" form

9. (5 points) Consider the following.

$$a_n = 3(n-5) \qquad \text{for } 1 \le n \le 10$$

stops at $n=10$

(a) Is this formula finite or infinite? finite / infinite (1)

(b) Is it a sequence or a series? sequence / series (1)

individual terms, not a sum

(c) Calculate the first three terms: -12, -9, -6

$a_1 = 3(1-5) = -12$

$a_2 = 3(2-5) = -9$

$a_3 = 3(3-5) = -6$

(1) each

10. (2 points) Label the following as "arithmetic", "geometric" or "neither".

(a) $-7, -4, -1, 2, \ldots$ add 3 arithmetic (1)

(b) $1, 1, 2, 3, 5, \ldots$ Fibonacci neither (1)

11. (3 points) Find the following sum, if it exists. If it does not exist, say so. If the answer requires scientific notation, give at least two decimal places. Show your work.

$$\sum_{k=0}^{42} 3^{k+2}$$

$k=0$ $k=1$ $k=2$

$= 3^2 + 3^3 + 3^4 + \ldots$

$= 9 + 27 + 81 + \ldots$

geometric with $r = 3$

$S_k = \frac{a_m(1-r^k)}{1-r}$

$= \frac{9(1-3^{43})}{1-3}$

$\approx 1.47716 \times 10^{21}$

$\approx 1.48 \times 10^{21}$

where $k = n - m + 1$

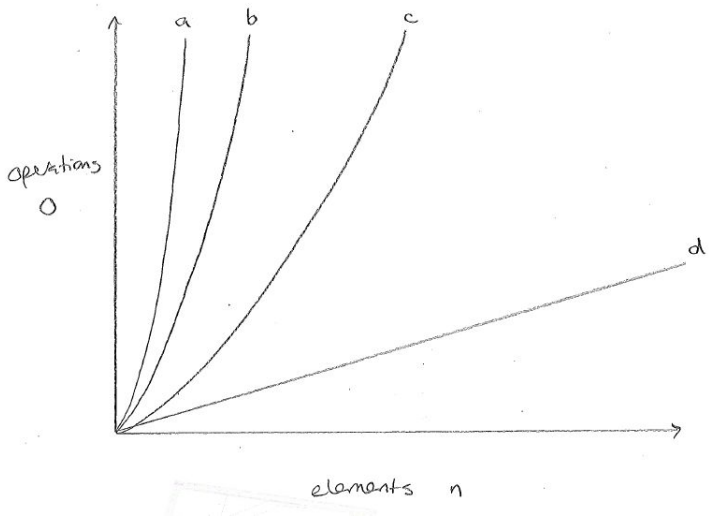
$= 42 - 0 + 1$

$= 43$

1. first few values correct or correct r
1. correct k
2. S_k

1.48×10^{21}

12. (4 points) Match the Big O notation with its corresponding curve on the graph. Please note that the curves are labeled a, b, c, and d going from left to right and that curve d is a straight line.



(a) $O(n^2)$ C

(b) $O(n!)$ a

(c) $O(n)$ d

(d) $O(2^n)$ b

13. (4 points) Consider the following variables concerning a statistics textbook.
    (a) The number of pages
    (b) The name of the publishing company
    (c) The price of the textbook
    (d) The mass of the textbook

    Which of these variables are qualitative? b (1)

    From the quantitative variables, which are discrete? a, c (2)

    (1) no incorrect answers

14. (4 points) The online video game store Steam is having its annual sale. Each game has a different price and you may assume that there are many games for sale.

    If every game has its price reduced by $15, what happens to the following quantities? Be as specific as you can!

    mean: reduced by $15

    standard deviation: stays same

15. (2 points) State whether the following study is experimental or observational by circling the correct choice.

    Civil engineers examined the statistics for the number of car accidents at a certain intersection in Victoria that used stop signs. They then turned the intersection into a roundabout and examined the statistics for the number of car accidents at that intersection afterwards. Expt / Observ

16. (4 points) The research team you are working with has taken a sample of six measurements. Unfortunately, the data file they sent you was corrupted and you were only able to extract the three lowest values. However, you do know that for this data set, the mean was 106.5, the median was 107, and the range was 19. Calculate the values of the three missing measurements, showing your work below.

data set is: 98, 101, 106, $\frac{108}{x}$, $\frac{109}{y}$, $\frac{117}{z}$

range = max - min

$19 = z - 98$

$z = 117$ (1)

median = average of two middle values

$107 = \frac{106 + x}{2}$

$214 = 106 + x$ (1)

$x = 108$

mean = $\frac{\sum x_i}{n}$

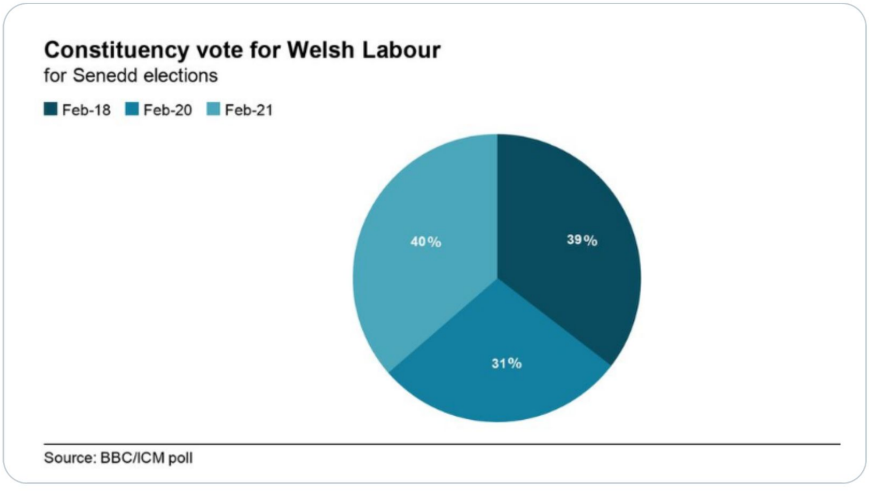
$106.5 = \frac{98 + 101 + 106 + 108 + y + 117}{6}$

$639 = y + 530$

$y = 109$ (2)

17. (2 points) The following graph was published by the BBC, regarding the results of an election for the Welsh Labour party. The percentages given for Feb 18, Feb 20, and Feb 21 are 39%, 31%, and 40%, respectively.

There is one <u>main</u> reason that this graph is badly designed. Give that reason.



The sum of the percentages does not equal 100% (it's 110%). (2)

[Note: this chart is not comparing parts of a single group, but rather the same part on different days. Should use bar chart.]

18. (4 points) How many 8-digit passwords are there if

(a) the passwords are alphanumeric but not case-sensitive?

36^8 or 2.82×10^{12}

_ _ _ _ _ _ _ _

each has $26 + 10 = 36$ choices

(b) the passwords are case-sensitive and alphanumeric and must contain at least one upper-case letter?

2.16×10^{14}

allowed = total − not allowed

$= 62^8 - 36^8$ ← no uppercase

$= 2.155 \times 10^{14}$

19. (4 points) A researcher for BC Ferries randomly selects a sample of sailings from the Vancouver - Victoria route and records whether the sailing departed on time or not and whether it was full. The results are displayed below.

	on time	late
full	12	3
not full	76	9

15

total = 100

In your answers below, **show enough work** that I can see which method you are using.

(a) What's the probability that a random sailing was full?

15%

$P(F) = \frac{n(F)}{n_{tot}} = \frac{15}{100}$

(b) What's the probability that a random sailing was full or late?

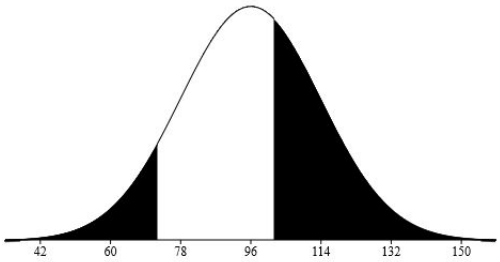
24%

$P(F \text{ or } L) = \frac{n(F \text{ or } L)}{n_{tot}} = \frac{12 + 3 + 9}{n_{tot}} = \frac{24}{100}$

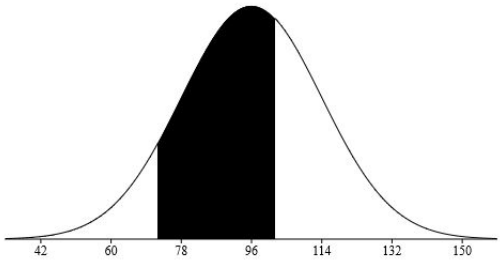
20. (2 points) According to the Mayo Clinic, the amount of caffeine in a standard cup of coffee is 96 mg. Let's assume that this amount of caffeine is normally distributed with a standard deviation of 18 mg.

    Suppose you wished to calculate the probability that a randomly chosen standard cup of coffee has between 72 and 102 mg of caffeine. Which of the following diagrams have shaded regions that correspond to that probability?

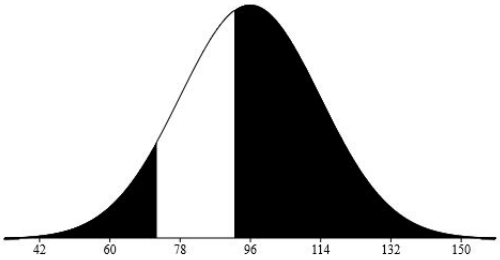
(a)



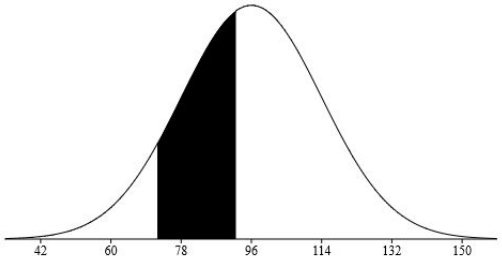
(b)



(c)



(d)



21. (5 points) A statistics instructor suspects that the average price of statistics textbooks has increased significantly in the past few years. She takes a random sample of 100 statistics textbooks and finds that they currently have an average price of \$202 with a standard deviation of \$47.

(a) Calculate an 90% confidence interval for the current average price of statistics textbooks. Show your work and round to the nearest dollar.

CI = \$194 to \$210

90% confidence gives $z = 1.645$ (1)

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

← large sample, so can use s for σ

$$= 202 \pm \frac{1.645(47)}{\sqrt{100}}$$

(3)

$$= 202 \pm 7.7315$$

$$= 202 \pm 8$$ (1)

so from \$194 to \$210 (1)

(b) What would happen to the confidence interval you calculated in part (a) if the statistics instructor increased the size of her sample? Briefly explain your answer.

the confidence interval would get narrower (smaller) because the margin of error has n in the denominator ($\sqrt{n}$, actually but same idea)

(1)