HOMEWORK PROBLEMS

Binary, Logic, and More

Applied Math for Computing

Patricia R. Wrean
Department of Mathematics & Statistics
Camosun College
wrean@camosun.ca

Exercises for Section 1.1

Consider the table below.

base 10	base 2	base 3	base 4	base 5	base 6	base 7	base 8
1			1_4				
2			2_4				
3			3_4				
4			10_4				
5			11_4				
6			12_4				
7			13_4				
8			20_4				
9			21_4				
10			22_4				
11			23_4				
12			30_4				
13							
14							
15							
16							
17							
18							
19							
20							

For the following exercises, complete the specified column in this table. The fourth column has been started as an example.

1. base 2
2. base 3
3. base 4
4. base 5

1.1 Exercises

5. base 6

6. base 7

7. base 8

In the number 12345678_{10}, in what place are the following digits?

8. 8

9. 6

10. 5

11. 7

12. 2

13. 1

In the number 1234567_8, which digit is in the

14. ones place?

15. eights place?

16. sixty-fours place?

17. 8^5 place?

The number 12345_8 can be expanded in base 10 as $1 \times 8^4 + 2 \times 8^3 + 3 \times 8^2 + 4 \times 8^1 + 5 \times 8^0$. Expand the following numbers into base 10 in a similar fashion.

18. 523_8

19. 1011110_2

20. 22013_4

21. 4130_5

22. 987_{10}

1.1 Exercises

Convert the following numbers to base 10:

23. 7231_8
24. 2031_4
25. 100_8
26. 1005_8
27. 2034_8

Answers to Section 1.1 Exercises

Here is the table for questions 1-7:

base 10	base 2	base 3	base 4	base 5	base 6	base 7	base 8
1	1_2	1_3	1_4	1_5	1_6	1_7	1_8
2	10_2	2_3	2_4	2_5	2_6	2_7	2_8
3	11_2	10_3	3_4	3_5	3_6	3_7	3_8
4	100_2	11_3	10_4	4_5	4_6	4_7	4_8
5	101_2	12_3	11_4	10_5	5_6	5_7	5_8
6	110_2	20_3	12_4	11_5	10_6	6_7	6_8
7	111_2	21_3	13_4	12_5	11_6	10_7	7_8
8	1000_2	22_3	20_4	13_5	12_6	11_7	10_8
9	1001_2	100_3	21_4	14_5	13_6	12_7	11_8
10	1010_2	101_3	22_4	20_5	14_6	13_7	12_8
11	1011_2	102_3	23_4	21_5	15_6	14_7	13_8
12	1100_2	110_3	30_4	22_5	20_6	15_7	14_8
13	1101_2	111_3	31_4	23_5	21_6	16_7	15_8
14	1110_2	112_3	32_4	24_5	22_6	20_7	16_8
15	1111_2	120_3	33_4	30_5	23_6	21_7	17_8
16	10000_2	121_3	100_4	31_5	24_6	22_7	20_8
17	10001_2	122_3	101_4	32_5	25_6	23_7	21_8
18	10010_2	200_3	102_4	33_5	30_6	24_7	22_8
19	10011_2	201_3	103_4	34_5	31_6	25_7	23_8
20	10100_2	202_3	110_4	40_5	32_6	26_7	24_8

8. ones
9. hundreds
10. thousands
11. tens

1.1 Answers

12. millions
13. ten millions
14. 7
15. 6
16. 5
17. 2
18. $523_8 = 5 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$
19. $1011110_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
20. $22013_4 = 2 \times 4^4 + 2 \times 4^3 + 0 \times 4^2 + 1 \times 4^1 + 3 \times 4^0$
21. $4130_5 = 4 \times 5^3 + 1 \times 5^2 + 3 \times 5^1 + 0 \times 5^0$
22. $987_{10} = 9 \times 10^2 + 8 \times 10^1 + 7 \times 10^0$
23. $7231_8 = 3737$
24. $2031_4 = 141$
25. $100_8 = 64$
26. $1005_8 = 517$
27. $2034_8 = 1052$

Exercises for Section 1.2

In the following binary numbers, in what place is the underlined number?

1. $1001010\underline{1}1$
2. $10010101\underline{1}$
3. $10\underline{0}101011$
4. $\underline{1}00101011$
5. $1001\underline{0}1011$

The number 11110_2 can be expanded in base 10 as $1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$. Expand the following numbers into base 10 in a similar fashion. Then perform that calculation to convert the number to base 10.

6. 10_2
7. 111_2
8. 1011_2
9. 1110111_2

Convert the following numbers to base 10.

10. 1001_2
11. 10110001_2
12. 10101_2

In the number $1C3D02_{16}$, in what place are the following digits?

13. 2
14. 0
15. D
16. 3
17. C
18. 1

The number 12345_{16} can be expanded in base 10 as $1 \times 16^4 + 2 \times 16^3 + 3 \times 16^2 + 4 \times 16^1 + 5 \times 16^0$. Expand the following numbers into base 10 in a similar fashion. You do not need to do the full calculation.

1.2 Exercises

19. 523_{16}
20. $F2_{16}$
21. $2A013_{16}$
22. $BEAD_{16}$
23. $9C8_{16}$

Convert the following numbers to base 10.

24. $AC882_{16}$
25. 1000_{16}
26. $2CF_{16}$
27. $BB8_{16}$
28. $7AAA01_{16}$
29. $65ABF_{16}$

Answers to Section 1.2 Exercises

1. the twos place
2. the ones place
3. the 64s place (2^6)
4. the 256s place (2^8)
5. the sixteens (2^4) place
6. $$\begin{aligned}10_2 &= 1 \times 2^1 + 0 \times 2^0 \\ &= 2 + 0 \\ &= 2\end{aligned}$$
7. $$\begin{aligned}111_2 &= 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 4 + 2 + 1 \\ &= 7\end{aligned}$$
8. $$\begin{aligned}1011_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 8 + 0 + 2 + 1 \\ &= 11\end{aligned}$$
9. $$\begin{aligned}1110111_2 &= 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 64 + 32 + 16 + 0 + 4 + 2 + 1 \\ &= 119\end{aligned}$$
10. $1001_2 = 9$
11. $10110001_2 = 177$
12. $10101_2 = 21$
13. ones
14. sixteens
15. 16^2
16. 16^3
17. 16^4
18. 16^5
19. $523_{16} = 5 \times 16^2 + 2 \times 16^1 + 3 \times 16^0$

1.2 Answers

20. $F2_{16} = 15 \times 16^1 + 2 \times 16^0$
21. $2A013_{16} = 2 \times 16^4 + 10 \times 16^3 + 0 \times 16^2 + 1 \times 16^1 + 3 \times 16^0$
22. $BEAD_{16} = 11 \times 16^3 + 14 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$
23. $9C8_{16} = 9 \times 16^2 + 12 \times 16^1 + 8 \times 16^0$
24. $AC882_{16} = 706690$
25. $1000_{16} = 4096$
26. $2CF_{16} = 719$
27. $BB8_{16} = 3000$
28. $7AAA01_{16} = 8038913$
29. $65ABF_{16} = 416447$

Exercises for Section 1.3

In the number 123.45678_{10}, in what place are the following digits?

1. 3
2. 6
3. 5
4. 7
5. 2
6. 1

In the number 1234.567_8, which digit is in the

7. ones place?
8. eighths place?
9. eights place?
10. sixty-fourths place?
11. sixty-fours place?

Convert the following numbers to base 10. When appropriate, round to 3 decimal places.

12. 72.31_8
13. 203.1_4
14. 100.111_2
15. 100.5_7
16. $20C4.B7_{16}$

Answers to Section 1.3 Exercises

1. ones
2. thousandths
3. hundredths
4. ten thousandths
5. tens
6. hundreds
7. 4
8. 5
9. 3
10. 6
11. 2
12. 58.391
13. 35.25
14. 4.875
15. 49.714
16. 8388.715

Exercises for Section 1.4

Convert the following decimal numbers to the indicated base.

1. 23 to octal
2. 12 to binary
3. 48 to hexadecimal

Convert the decimal number 1234 to the following bases.

4. binary
5. octal
6. hexadecimal
7. base 7

Convert the following decimal numbers to the indicated base.

8. 7203 to octal
9. 123 to binary
10. 11331 to hexadecimal

Perform the following conversions for non-integer numbers. Give exact answers (do not round off).

11. 0.359375 to octal
12. 0.8125 to binary
13. 0.234375 to hexadecimal

Perform the following conversions for non-integer numbers. Use the repeater bar in your answer.

14. 0.6 to octal
15. 0.3 to binary
16. 0.36 to hexadecimal

Perform the following conversions. Give exact answers (do not round off).

17. 18.125 to hexadecimal
18. 31.6 to base 4

1.4 Exercises

19. 37.875 to octal
20. 23.35 to binary

Answers to Section 1.4 Exercises

1. $23 = 27_8$
2. $12 = 1100_2$
3. $48 = 30_{16}$
4. $1234 = 10011010010_2$
5. $1234 = 2322_8$
6. $1234 = 4D2_{16}$
7. $1234 = 3412_7$
8. $7203 = 16043_8$
9. $123 = 1111011_2$
10. $11331 = 2C43_{16}$
11. 0.27_8
12. 0.1101_2
13. $0.3C_{16}$
14. $0.\overline{4631}_8$
15. $0.0\overline{1001}_2$ (if you don't notice the repeating pattern immediately, the answers $0.01\overline{0011}_2$, $0.010\overline{0110}_2$, etc., are also acceptable)
16. $0.\overline{5C28F}_{16}$
17. 12.2_{16}
18. $133.\overline{21}_4$
19. 45.7_8
20. $10111.01\overline{0110}_2$

Exercises for Section 1.5

Convert the following octal numbers to binary:

1. 113_8
2. 20.1_8
3. 1104_8

Convert the following hexadecimal numbers to binary:

4. $2B_{16}$
5. $3C.C_{16}$
6. $29A_{16}$

Convert the following binary numbers to octal:

7. 1100_2
8. 1001100_2
9. 11011.1001_2

Convert the following binary numbers to hexadecimal:

10. 10011_2
11. 1000000_2
12. 1.101111_2

Convert the following octal numbers to hexadecimal:

13. 1.6_8
14. 142_8
15. 24.57_8
16. 5002_8

Convert the following hexadecimal numbers to octal:

17. $C.2_{16}$
18. $1D07_{16}$
19. $A.2E6_{16}$

1.5 Exercises

Perform the following conversions for non-integer numbers:

20. $E.15_{16}$ to binary
21. 4.702_8 to binary
22. 10.011_2 to hexadecimal
23. 110.1_2 to octal
24. $7B.B_{16}$ to octal
25. 4.1702_8 to hexadecimal

Answers to Section 1.5 Exercises

1. $113_8 = 1001011_2$
2. $20.1_8 = 10\ 000.001_2$
3. $1104_8 = 1001000100_2$
4. $2B_{16} = 101011_2$
5. $3C.C_{16} = 11\ 1100.11_2$
6. $29A_{16} = 1010011010_2$
7. $1100_2 = 14_8$
8. $1001100_2 = 114_8$
9. $11011.1001_2 = 33.44_8$
10. $10011_2 = 13_{16}$
11. $1000000_2 = 40_{16}$
12. $1.101111_2 = 1.BC_{16}$
13. $1.6_8 = 1.C_{16}$
14. $142_8 = 62_{16}$
15. $24.57_8 = 14.BC_{16}$
16. $5002_8 = A02_{16}$
17. $C.2_{16} = 14.1_8$
18. $1D07_{16} = 16407_8$
19. $A.2E6_{16} = 12.1346_8$
20. $E.15_{16} = 1110.0001\ 0101_2$
21. $4.702_8 = 100.111\ 000\ 01_2$
22. $10.011_2 = 2.6_{16}$
23. $110.1_2 = 6.4_8$
24. $7B.B_{16} = 173.54_8$
25. $4.1702_8 = 4.3C2_{16}$

Ch 1

Mixed Practice

Convert the following numbers to the indicated base. Give exact answers unless directed otherwise. Show your work.

1. $5B2_{16}$ to binary
2. 0.12_{16} to decimal
3. 5392 to octal
4. 19.5625 to binary
5. 11010.01011_2 to octal
6. 703.1_8 to decimal
7. 0.33 to hexadecimal
8. 33.72_8 to hexadecimal
9. 44.02_5 to decimal
10. 101010.01_2 to hexadecimal
11. 44.02 to base 5
12. 262.8125 to octal

Ch 1
Mixed Practice

Answers

1. $101\ 1011\ 0010_2$ (the spacing is not necessary, but it makes the result easier to read)
2. 0.0703125
3. 12420_8
4. 10011.1001_2
5. 32.26_8
6. 451.125
7. $0.5\overline{47AE1}_{16}$
8. $1B.E8_{16}$
9. 24.08
10. $2A.4_{16}$
11. $134.00\overline{2}_5$
12. 406.64_8

Exercises for Section 2.1

State whether the following sentences are propositions.

1. On September 6, 2006, mathematicians proved that $2^{32582657} - 1$ was a prime number.

2. Will you marry me?

3. Python is her favourite computing language.

4. What is your favourite computing language?

5. Please bring me a textbook.

6. The University of Victoria is located in Alberta.

Let p be "Rich is seven feet tall" and q be "Susan has brown hair." Translate the following English sentences into logical notation.

7. Rich is seven feet tall or he is seven feet tall.

8. Either Rich is not seven feet tall or Susan does not have brown hair.

9. It is not true that Rich is seven feet tall or Susan has brown hair.

10. Rich is seven feet tall and Susan has brown hair.

11. Either Rich is seven feet tall or Susan does not have brown hair, but not both.

Which type of "or", inclusive or exclusive, is meant in the following English sentences?

12. Do you want to sit inside or outside?

13. Have you seen the latest Harry Potter or Transformers movie?

14. I think I'll get an A or a B in the course.

15. Is that the correct answer or not?

16. We need someone who speaks French or German.

2.1 Exercises

Let p be "The moon is made of green cheese" and q be "The earth is made of green cheese." Translate the following English sentences into logical notation.

17. Either the moon is made of green cheese or both the moon and the earth are made of green cheese.
18. The earth is made of green cheese and either the earth or the moon is made of green cheese.
19. Either the earth is made of green cheese while the moon is not, or the moon is made of green cheese.
20. The earth is made of green cheese and either the moon is made of green cheese or the earth is not.

Let $p =$ "Jane did her homework" and $q =$ "Jane went for a jog." Translate the following logical propositions into English sentences.

21. $p \wedge q$
22. $\sim(p \wedge q)$
23. $q \wedge \sim p$
24. $\sim q \vee \sim p$
25. $\sim(\sim p)$ (that's "not(not p)")
26. $q \oplus \sim q$

For each pair of sentences below, is the second sentence the negation of the first?

27. Pat owes Peter money. Peter owes Pat money.
28. The number of students in Math 155 is greater than 25. The number of students in Math 155 is less than 25.
29. Pat, the math instructor, is rich. Pat, the math instructor, is poor.

Answer the questions given the following situations. If you cannot answer the question, state whether "the situation is not possible" or "there's not enough information."

30. Jane went for a jog and did her homework. Did she go for a jog?
31. Jane went for a jog or did her homework. Did she not do her homework?

2.1 Exercises

32. Jane went for a jog. Did she go for a jog and do her homework?
33. Jane did not go for a jog. Did she go for a jog and do her homework?

Answers to Section 2.1 Exercises

1. Yes
2. No
3. No
4. No
5. No
6. Yes
7. $p \vee p$
8. From the context, you could go with either $\sim p \vee \sim q$ or $\sim p \oplus \sim q$.
9. $\sim(p \vee q)$
10. $p \wedge q$
11. $p \oplus \sim q$
12. exclusive (you usually don't sit both inside and outside at the same time)
13. inclusive (you could have seen both)
14. exclusive (you can only get one mark for the course, so it's one or the other but can't be both)
15. exclusive (it can't both be the correct answer and not the correct answer at the same time)
16. inclusive (it's possible that someone speaks both languages)
17. $p \vee (p \wedge q)$
18. $q \wedge (q \vee p)$
19. $(q \wedge \sim p) \vee p$
20. $q \wedge (p \vee \sim q)$
21. Jane did her homework and went for a jog.
22. It is not true that Jane both did her homework and went for a jog.
23. Jane went for a jog and Jane did not do her homework.

2.1 Answers

24. Jane did not go for a jog or she didn't do her homework.
25. It is not true that Jane didn't do her homework.
26. Either Jane went for a jog or she didn't, but not both.
27. No. (They could just be even, not owing each other anything.)
28. No. (What if there were exactly 25 students in the class?)
29. No. (Maybe Pat is middle class, so is neither rich nor poor?)
30. Yes.
31. Not enough info. Depends on whether she went for a jog. If she did go for a jog, she could have not done her homework. But if she didn't go for a jog, she must have done her homework for sure.
32. Not enough info. Depends on whether she did her homework.
33. No.

Exercises for Section 2.2

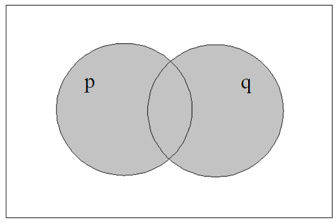
Draw Venn diagrams using two propositions p and q, shading in the appropriate regions for the following situations.

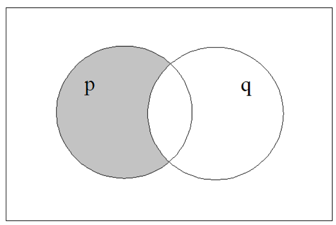
1. $p \vee q$
2. $p \wedge \sim q$
3. $\sim p \wedge \sim q$
4. $\sim(p \wedge \sim q)$ (this would just be the negation of #2)
5. $\sim(p \vee q)$
6. $p \wedge (\sim p \vee q)$
7. $p \vee (p \wedge q)$

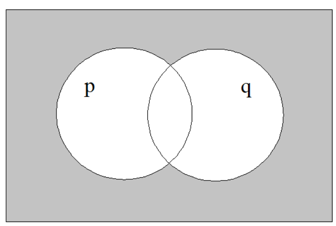
Draw Venn diagrams using three propositions: p, q, and r. Shade in the appropriate regions for the following situations.

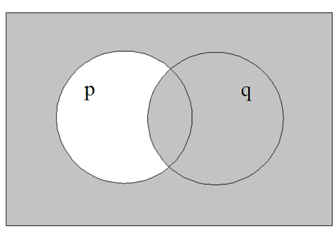
8. $p \vee q \vee r$
9. $(p \wedge q) \vee r)$
10. $p \wedge (q \vee r)$
11. $p \vee \sim q \vee r$
12. $\sim p \wedge q \wedge \sim r$
13. $(p \wedge q) \vee \sim r$
14. $\sim q \wedge (\sim p \vee r)$

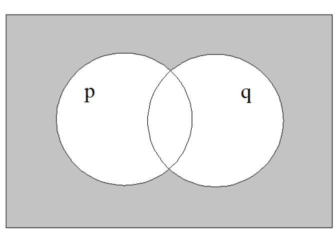
Answers to Section 2.2 Exercises

1. 

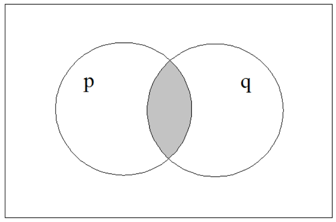
2. 

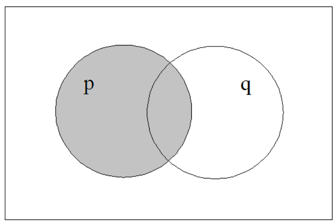
3. 

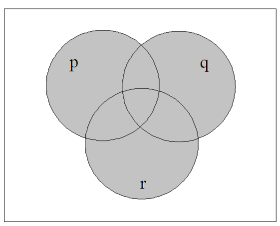
4. 

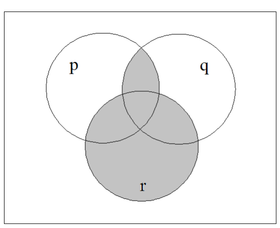
5. 

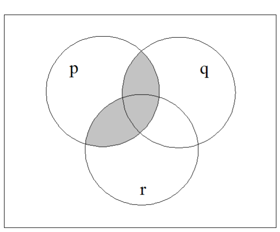
2.2 Answers

6. 

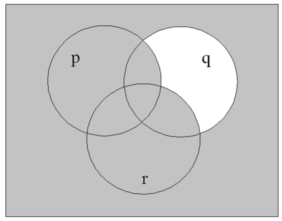
7. 

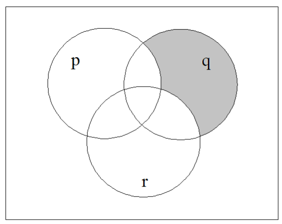
8. 

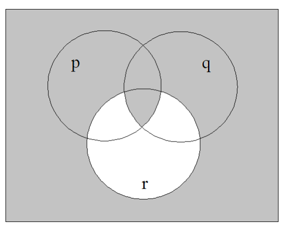
9. 

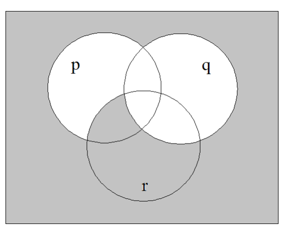
10. 

2.2 Answers

11. 

12. 

13. 

14. 

Exercises for Section 2.3

Give the truth tables for the following logical expressions.

1. $p \wedge \sim p$
2. $p \vee 1$
3. $p \wedge \sim q$
4. $\sim(p \vee q)$
5. $p \oplus \sim q$
6. $p \vee (\sim p \wedge q)$
7. $(p \vee q) \wedge r$
8. $p \vee q \vee \sim r$
9. $(p \wedge q) \vee \sim(p \vee \sim q)$
10. $(\sim p \vee \sim q) \wedge (\sim p \vee q)$

Are the two expressions logically equivalent?

11. $\sim(p \wedge q)$ and $\sim p \wedge \sim q$
12. $\sim(p \vee q)$ and $\sim p \wedge \sim q$
13. $p \oplus q$ and $\sim p \oplus \sim q$
14. $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$
15. $p \vee (p \wedge q)$ and p
16. $(p \vee q) \vee r$ and $p \vee (q \vee r)$
17. $p \oplus q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$

Simplify.

18. $p \wedge p$
19. $p \vee \sim p$
20. $p \wedge 0$
21. $\sim p \oplus p$
22. $(p \oplus q) \wedge (p \oplus \sim q)$

2.3 Exercises

23. $p \vee (p \wedge q)$
24. $q \wedge (p \vee q)$
25. (tricksy) $p \wedge (\sim p \vee q)$
26. (tricksy) $p \vee (\sim p \wedge q)$

Answers to Section 2.3 Exercises

1.

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

2.

p	1	$p \vee 1$
0	1	1
1	1	1

3.

p	q	$\sim q$	$p \wedge \sim q$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

4.

p	q	$p \vee q$	$\sim(p \vee q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

5.

p	q	$\sim q$	$p \oplus \sim q$
0	0	1	1
0	1	0	0
1	0	1	0
1	1	0	1

2.3 Answers

6.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

7.

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

8.

p	q	r	$\sim r$	$p \vee q \vee \sim r$
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	1
1	1	0	1	1
1	1	1	0	1

2.3 Answers

9.

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$(p \wedge q) \vee \sim(p \vee \sim q)$
0	0	1	0	1	0	0
0	1	0	0	0	1	1
1	0	1	0	1	0	0
1	1	0	1	1	0	1

10.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim p \vee q$	$(\sim p \vee \sim q) \wedge (\sim p \vee q)$
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	1	0	0
1	1	0	0	0	1	0

11.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	0	1	1	0	0
1	0	0	1	0	1	0
1	1	1	0	0	0	0

No, because the 4th and 7th columns are not the same.

12.

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Yes, because the 4th and 7th columns are identical.

2.3 Answers

13.

p	q	$p \oplus q$	$\sim p$	$\sim q$	$\sim p \oplus \sim q$
0	0	0	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

Yes, because the 3rd and 6th columns are identical.

14.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	0
1	0	1	0	1	1	1
1	1	0	0	1	1	0
1	1	1	1	1	1	1

No, because the 5th and last columns are not identical.

15.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Yes, because the first and last columns are identical.

2.3 Answers

16.

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Yes, because the 5th and last columns are identical.

17.

p	q	$p \oplus q$	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
0	0	0	1	1	0	1	1
0	1	1	1	0	0	0	0
1	0	1	0	1	0	0	0
1	1	0	0	0	1	0	1

No, because the 3rd and last columns are not identical. (But I think you can see that the last expression is the negation of column 3.)

18.

p	p	$p \wedge p$
0	0	0
1	1	1

This expression is logically equivalent to p. (You can omit the second column for p if you wish.)

19.

p	$\sim p$	$p \vee \sim p$
0	1	1
1	0	1

This expression is logically equivalent to 1.

2.3 Answers

20.

p	0	$p \wedge 0$
0	0	0
1	0	0

This expression is logically equivalent to 0.

21.

p	$\sim p$	$\sim p \oplus p$
0	1	1
1	0	1

This expression simplifies to 1.

22.

p	q	$p \oplus q$	$\sim q$	$p \oplus \sim q$	$(p \oplus q) \wedge (p \oplus \sim q)$
0	0	0	1	1	0
0	1	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	0

This expression simplifies to 0.

23.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

This expression is logically equivalent to p.

24.

p	q	$p \vee q$	$q \wedge (p \vee q)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	1	1

2.3 Answers

This expression simplifies to q.

25.

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

This expression is logically equivalent to $p \wedge q$.

26.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

This expression simplifies to $p \vee q$.

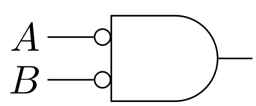
Exercises for Section 2.4

Draw the gate representation for the following logical expressions.

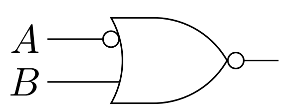
1. $A + \overline{B}$
2. $\overline{A + B}$
3. $\overline{A}B$
4. $\overline{A}\ \overline{B}$
5. $\overline{A + \overline{\overline{B}}}$
6. $A\overline{B} + C$
7. $A(B + \overline{C})$
8. $\overline{AB}C$
9. $\overline{\overline{\overline{A}\ \overline{B}} + \overline{C}}$

Write the Boolean expression which corresponds to the following gates.

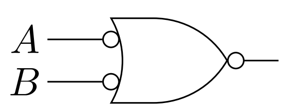
10.



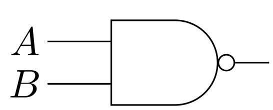
11.



12.

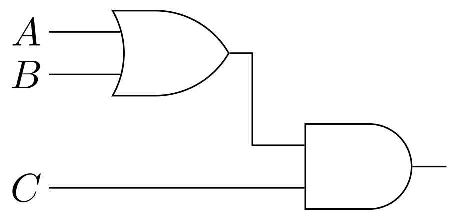


13.

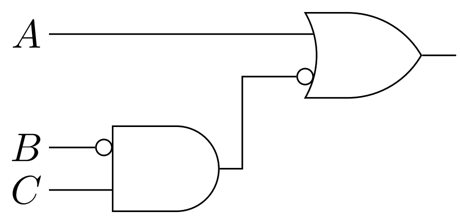


2.4 Exercises

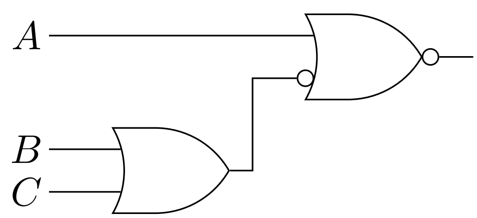
14.



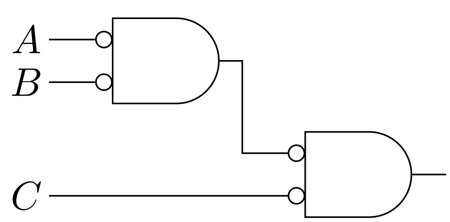
15.



16.



17.



Give the truth tables for the following expressions.

18. $A\overline{A}$

19. $A + 1$

20. $A\overline{B}$

2.4 Exercises

21. $\overline{A+B}$
22. $A+\overline{A}B$
23. $(A+B)C$
24. $A+B+\overline{C}$

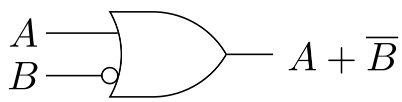
Are the two expressions logically equivalent? Justify your answer by giving a truth table.

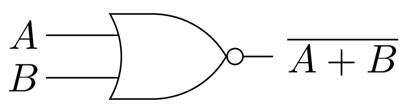
25. $\overline{AB}$ and $\overline{A}\ \overline{B}$
26. $\overline{A+B}$ and $\overline{A}\ \overline{B}$
27. $A+BC$ and $(A+B)C$
28. $A+AB$ and A
29. $(A+B)+C$ and $A+(B+C)$

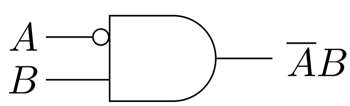
Simplify the following logical expressions using truth tables.

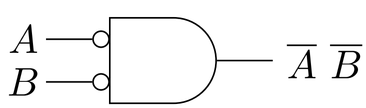
30. AA
31. $A+A$
32. $A+0$
33. $A+AB$
34. $A(\overline{A}+B)$ – this one's a bit trickier! If you're stuck, try writing the truth tables for combinations of A and B, like $(A+B)$ for example, to find one that fits.

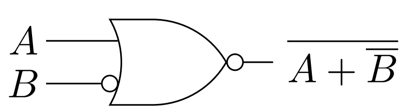
Answers to Section 2.4 Exercises

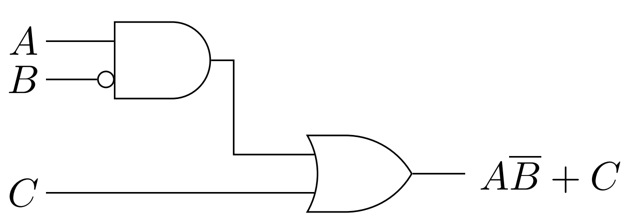
1. 

2. 

3. 

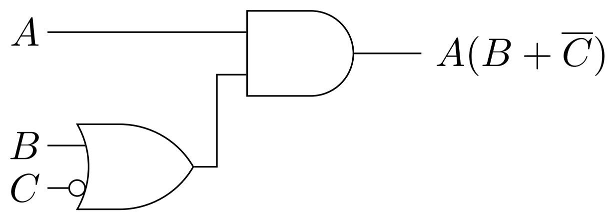
4. 

5. 

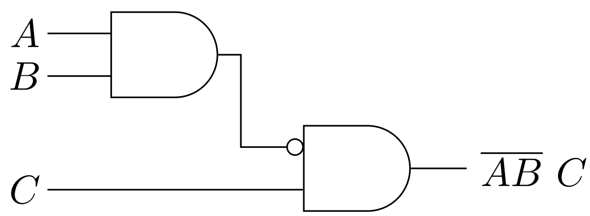
6. 

2.4 Answers

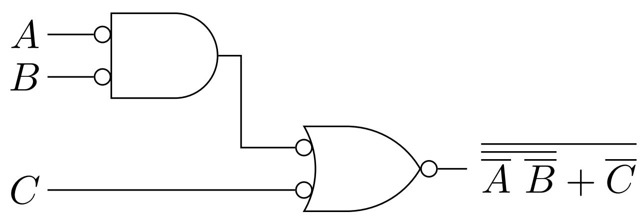
7.



8.



9.



10. $\overline{A}\ \overline{B}$

11. $\overline{\overline{A} + B}$

12. $\overline{\overline{A} + \overline{B}}$

13. $\overline{AB}$

14. $(A + B) \cdot C$

15. $A + \overline{\overline{B}C}$

16. $\overline{A + \overline{B + C}}$

17. $\overline{\overline{A}\ \overline{B}}\ \overline{C}$

2.4 Answers

18.

A	$\overline{A}$	$A\,\overline{A}$
0	1	0
1	0	0

19.

A	1	$A+1$
0	1	1
1	1	1

20.

A	B	$\overline{B}$	$A\,\overline{B}$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

21.

A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

22.

A	B	$\overline{A}$	$\overline{A}\,B$	$A+\overline{A}\,B$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

2.4 Answers

23.

A	B	C	$A+B$	$(A+B)C$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

24.

A	B	C	$\overline{C}$	$A+B$	$A+B+\overline{C}$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	0	1	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1

25. No

A	B	AB	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A}\ \overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	0
1	0	0	1	0	1	0
1	1	1	0	0	0	0

26. Yes

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\ \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

2.4 Answers

27. No

A	B	C	BC	$A+BC$	$A+B$	$(A+B)C$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	0
1	0	1	0	1	1	1
1	1	0	0	1	1	0
1	1	1	1	1	1	1

28. Yes

A	B	AB	$A+AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

29. Yes

A	B	C	$A+B$	$(A+B)+C$	$B+C$	$A+(B+C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

30.

A	A	AA
0	0	0
1	1	1

AA is equivalent to A

2.4 Answers

31.

A	A	$A+A$
0	0	0
1	1	1

$A+A$ is equivalent to A

32.

A	0	$A+0$
0	0	0
1	0	1

$A+0$ is equivalent to A

33.

A	B	AB	$A+AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$A+AB$ is equivalent to A

34.

A	B	$\overline{A}$	$\overline{A}+B$	$A(\overline{A}+B)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

$A(\overline{A}+B)$ is logically equivalent to AB

Exercises for Section 2.5

1. Which of the following statements is always true?
   (a) Darth Vader is both evil and not evil.
   (b) Darth Vader is both evil and evil.
   (c) Darth Vader is either evil or evil.
   (d) Darth Vader is either evil or not evil.
2. Which of the following statements is always false?
   (a) The roadrunner has escaped from the wily coyote and he has not escaped from the wily coyote.
   (b) The roadrunner has escaped from the wily coyote and he has escaped from the wily coyote.
   (c) The roadrunner has escaped from the wily coyote or he has not escaped from the wily coyote.
   (d) The roadrunner has escaped from the wily coyote or he has escaped from the wily coyote.
3. Use a truth table to prove that the two idempotent laws are true.
4. Use a truth table to prove that the four identity laws are true.

Name the law of logic used in the following. Note that the variables have changed, but that the law is still valid.

5. $\sim q \vee 1 \Leftrightarrow 1$
6. $\overline{\overline{B}} = B$
7. $\sim r \wedge r \Leftrightarrow 0$
8. $\sim q \vee 0 \Leftrightarrow \sim q$
9. $\overline{B} \cdot 1 = \overline{B}$
10. $q \vee q \Leftrightarrow q$
11. $AB + \overline{AB} = 1$
12. $(\sim p \wedge q) \wedge \sim q \Leftrightarrow \sim p \wedge (q \wedge \sim q)$

2.5 Exercises

Simplify the given expression, and state the name of the law you used. You should be able to do these in one step.

13. $r \vee 0$
14. $C + \overline{C}$
15. $\sim(\sim r)$
16. $\overline{A} + \overline{A}$
17. $\overline{B} \cdot 1$

Use the laws of logic to simplify the following logical expressions. If you're completely stuck, try using a truth table instead.

18. $(p \wedge p) \vee (q \wedge \sim q)$
19. $(p \vee p) \wedge (q \vee 0)$
20. $p \vee (q \wedge \sim q)$

Use the laws of logic to simplify the following Boolean expressions. If you're completely stuck, try using a truth table instead.

21. $(A + A)(B + \overline{B})$
22. $B \cdot 0 + AA$
23. $(B + \overline{B})(A + 1)$
24. $AB\overline{B}$

Prove the following Boolean expressions are equivalent using the laws of logic. If you're completely stuck, try using a truth table.

25. $(A\overline{A})\overline{B} = A(B\overline{B})$
26. $B \cdot 1 + A\overline{A} = \overline{\overline{B} \cdot 1}$
27. $(A + 0)(B + \overline{B}) = A$
28. $AA + \overline{B}\ \overline{B} = A + \overline{B}$

Answers to Section 2.5 Exercises

1. (d) is true because in logical symbols, $p \vee \sim p \Leftrightarrow 1$.

2. (a) is false because $p \wedge \sim p \Leftrightarrow 0$.

3. The two idempotent laws are true because the last column in each table is the same as for p.

p	p	$p \vee p$
0	0	0
1	1	1

p	p	$p \wedge p$
0	0	0
1	1	1

4. The four identity laws are true because the $p \wedge 0$ column is the same as 0, the $p \vee 0$ and $p \wedge 1$ columns are the same as p, and the $p \vee 1$ column is the same as 1.

p	0	1	$p \wedge 0$	$p \vee 0$	$p \wedge 1$	$p \vee 1$
0	0	1	0	0	0	1
1	0	1	0	1	1	1

5. identity

6. complement

7. complement

8. identity

9. identity

10. idempotent

11. complement

12. associative

13. r, using the identity law

14. 1, complement

15. r, complement

16. $\overline{A}$, idempotent

17. $\overline{B}$, identity

2.5 Answers

Note: for the following questions, there may be several different ways to get to the simplest answer. Also, you may take steps in a different order. If you are concerned about a different solution, please show your instructor. (Also, I haven't explicitly written out any steps involving either the Commutative or Associative laws.)

18. $(p \wedge p) \vee (q \wedge {\sim}q) \Leftrightarrow p \vee (q \wedge {\sim}q)$ Idempotent
    $\Leftrightarrow p \vee 0$ Complement
    $\Leftrightarrow p$ Identity

19. $(p \vee p) \wedge (q \vee 0) \Leftrightarrow p \wedge (q \vee 0)$ Idempotent
    $\Leftrightarrow p \wedge q$ Identity

20. $p \vee (q \wedge {\sim}q) \Leftrightarrow p \vee 0$ Complement
    $\Leftrightarrow p$ Identity

21. $(A + A)(B + \overline{B}) = A(B + \overline{B})$ Idempotent
    $= A \cdot 1$ Complement
    $= A$ Identity

22. $B \cdot 0 + AA = 0 + AA$ Identity
    $= 0 + A$ Idempotent
    $= A$ Identity

23. $(B + \overline{B})(A + 1) = 1 \cdot (A + 1)$ Complement
    $= 1 \cdot 1$ Identity
    $= 1$ Definition of "and"

24. $AB\overline{B} = A \cdot 0$ Complement
    $= 0$ Identity

25. $(A\overline{A})\overline{B} = A(B\overline{B})$
    $0 \cdot \overline{B} = A(B\overline{B})$ Complement
    $0 \cdot \overline{B} = A \cdot 0$ Complement
    $0 = A \cdot 0$ Identity
    $0 = 0$ Identity

2.5 Answers

26.
$$\begin{aligned}
B \cdot 1 + A\overline{A} &= \overline{\overline{B} \cdot 1} \\
B + A\overline{A} &= \overline{\overline{B}} && \text{Identity} \\
B + 0 &= \overline{\overline{B}} && \text{Complement} \\
B &= \overline{\overline{B}} && \text{Identity} \\
B &= B && \text{Complement}
\end{aligned}$$

27.
$$\begin{aligned}
(A + 0)(B + \overline{B}) &= A \\
A(B + \overline{B}) &= A && \text{Identity} \\
A \cdot 1 &= A && \text{Complement} \\
A &= A && \text{Identity}
\end{aligned}$$

28.
$$\begin{aligned}
AA + \overline{B}\ \overline{B} &= A + \overline{B} \\
A + \overline{B} &= A + \overline{B} && \text{Idempotent}
\end{aligned}$$

Exercises for Section 2.6

(Note that these are the same exercises as at the beginning of section 1.5, but with a little twist.) Let p be "Rich is seven feet tall" and q be "Susan has brown hair." Translate the following English sentences into logical notation. Then, use one of the laws of logic to write an equivalent logical expression. Finally, translate your new expression back into an English sentence.

1. Rich is seven feet tall or he is seven feet tall.
2. Susan has brown hair and she has brown hair.
3. Either Rich is not seven feet tall or Susan does not have brown hair.
4. It is not true that Rich is seven feet tall and Susan has brown hair.
5. It is not true that Rich is seven feet tall or Susan has brown hair.
6. Rich is not seven feet tall and Susan does not have brown hair.
7. Rich is seven feet tall and Susan has brown hair.
8. Susan has brown hair or Rich is seven feet tall.

Name the law of logic used in the following. Note that the variables have changed, but that the law is still valid.

9. $\sim(q \vee r) \Leftrightarrow \sim q \wedge \sim r$
10. $\overline{B}(B+\overline{A}) = \overline{B}\ \overline{A}$
11. $(p \wedge q) \vee (p \wedge \sim q) \Leftrightarrow p \wedge (q \vee \sim q)$
12. $\overline{\overline{A}+C} = A\overline{C}$
13. $B + A\overline{C} = (B+A)(B+\overline{C})$
14. $\sim p \vee (p \wedge r) \Leftrightarrow \sim p \vee r$

Simplify the given expression, and state the name of the law you used. You should be able to do these in a single step.

15. $\overline{A} + A\overline{B}$
16. $\overline{AB} + \overline{AB}$
17. $(A+B)(B+C)$
18. $q \vee (q \wedge r)$

2.6 Exercises

19. $\overline{C} + C$

20. $\overline{\overline{A}\;\overline{B}}$

For the following exercises, let p be "The moon is made of green cheese" and q be "The earth is made of green cheese." Translate the following English sentences into logical notation. Then, use one of the laws of logic to write an equivalent logical expression. Finally, translate your new expression back into an English sentence. (Note that these are the same exercises as at the beginning of section 2.1, but with a little twist.)

21. Either the moon is made of green cheese or both the moon and the earth are made of green cheese.

22. The earth is made of green cheese and either the earth or the moon is made of green cheese.

23. Either the earth is made of green cheese while the moon is not, or the moon is made of green cheese.

24. The earth is made of green cheese and either the moon is made of green cheese or the earth is not.

25. Remembering that $\oplus$ is "exclusive or", show that $\mathrm{A}\oplus\mathrm{B} = \overline{A}\mathrm{B} + \mathrm{A}\overline{B}$ by using a truth table.

26. The `NAND` gate (not-AND) has the following truth table. Use DeMorgan's laws to find an equivalent Boolean expression using only OR and NOT, and show that your expression has the same truth table.

A	B	A `NAND` $B = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

Simplify the following Boolean expressions using the laws of logic. If you're stuck, try using a truth table.

27. $A + \overline{C} + B + \overline{A} + \overline{B}$

28. $A + \overline{B} + A + B + A$

29. $\overline{\overline{A}\;\overline{B}}$

2.6 Exercises

30. $\overline{\overline{A} + \overline{B}}$
31. $\overline{A} + B + A\overline{B}$
32. $A\overline{B}\,\overline{C} + A\overline{B}C$
33. $\overline{A}BC + \overline{A}B\overline{C} + \overline{A}B\overline{D} + \overline{A}BD$
34. $AB + A + \overline{AB}$
35. $A + \overline{B}CD + \overline{B}$
36. $\overline{A}\,\overline{B}(A + B)$
37. $(\overline{A} + \overline{B})(A + B)$
38. $A + \overline{A}B + \overline{B}C$
39. $B(A + C) + \overline{A}B\overline{C}$
40. $(A + B + C)(A + B + \overline{C})$

Prove that the following Boolean expressions are equivalent by using the laws of logic. If you're stuck, try using a truth table.

41. $B\overline{B} + AA = A$
42. $\overline{A}(B + \overline{B}) = \overline{A}$
43. $ABC + AB\overline{C} = AB$
44. $AB + \overline{AB}C = AB + C$
45. $A + AB + ABC = A$
46. $\overline{A}C + A\overline{B}C = \overline{A}C + \overline{B}C$
47. $\overline{AB}(A + B) = \overline{A}B + A\overline{B}$
48. $\overline{\overline{\overline{A}BC} + D} = \overline{A}BC\overline{D}$
49. $A\,\overline{B}\,\overline{\overline{A}\,\overline{C}} = A\,\overline{B}$

Answers to Section 2.6 Exercises

1. $p \vee p \Leftrightarrow p$. Rich is seven feet tall.
2. $q \wedge q \Leftrightarrow q$. Susan has brown hair.
3. $\sim p \vee \sim q \Leftrightarrow \sim (p \wedge q)$. It is not the case that Rich is seven feet tall and Susan has brown hair.
4. $\sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$. Rich is not seven feet tall or Susan does not have brown hair.
5. $\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$. Rich is not seven feet tall and Susan does not have brown hair.
6. $\sim p \wedge \sim q \Leftrightarrow \sim (p \vee q)$. It is not the case that Rich is seven feet tall or Susan has brown hair.
7. $p \wedge q \Leftrightarrow q \wedge p$. Susan has brown hair and Rich is seven feet tall.
8. $q \vee p \Leftrightarrow p \vee q$. Rich is seven feet tall or Susan has brown hair.
9. De Morgan's
10. absorption
11. distributive
12. De Morgan's
13. distributive
14. absorption
15. $\overline{A} + \overline{B}$, absorption
16. $\overline{AB}$, idempotent
17. $B + AC$, distributive
18. q, absorption
19. 1, complement
20. $A + B$, De Morgan's
21. $p \vee (p \wedge q) \Leftrightarrow p$. The moon is made of green cheese.
22. $q \wedge (q \vee p) \Leftrightarrow q$. The earth is made of green cheese.

2.6 Answers

23. $(q \wedge \sim p) \vee p \Leftrightarrow p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$. (note: I'm using the commutative laws to rearrange things) The moon or the earth is made of green cheese.

24. $q \wedge (p \vee \sim q) \Leftrightarrow q \wedge p$. The earth and the moon are made of green cheese.

25.

A	B	$A \oplus B$	$\overline{A}$	$\overline{B}$	$\overline{A}B$	$A\overline{B}$	$\overline{A}B + A\overline{B}$
0	0	0	1	1	0	0	0
0	1	1	1	0	1	0	1
1	0	1	0	1	0	1	1
1	1	0	0	0	0	0	0

26. By DeMorgan's law, $\overline{AB} = \overline{A} + \overline{B}$

A	B	A NAND $B = \overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

27. 1

28. 1

29. $A + B$

30. AB

31. 1

32. $A\overline{B}$

33. $\overline{A}B$

34. 1

35. $A + \overline{B}$

36. 0

37. $A\overline{B} + \overline{A}B$

38. $A + B + C$

2.6 Answers

39. B

40. $A+B$

41.
$$\begin{aligned} B\overline{B}+AA &= A \\ 0+AA &= A \quad \text{complement} \\ AA &= A \quad \text{identity} \\ A &= A \quad \text{idempotent} \end{aligned}$$

42.
$$\begin{aligned} \overline{A}(B+\overline{B}) &= \overline{A} \\ \overline{A}(1) &= \overline{A} \quad \text{complement} \\ \overline{A} &= \overline{A} \quad \text{identity} \end{aligned}$$

43.
$$\begin{aligned} ABC+AB\overline{C} &= AB \\ AB(C+\overline{C}) &= AB \quad \text{distributive} \\ AB(1) &= AB \quad \text{complement} \\ AB &= AB \quad \text{identity} \end{aligned}$$

44.
$$\begin{aligned} AB+\overline{AB}C &= AB+C \\ (AB)+(\overline{AB})C &= AB+C \quad \text{associative (can skip this step)} \\ AB+C &= AB+C \quad \text{absorption} \end{aligned}$$

45.
$$\begin{aligned} A+AB+ABC &= A \\ A+ABC &= A \quad \text{absorption} \\ A+A(BC) &= A \quad \text{associative (can skip)} \\ A &= A \quad \text{absorption} \end{aligned}$$

46.
$$\begin{aligned} \overline{A}C+A\overline{B}C &= \overline{A}C+\overline{B}C \\ (\overline{A}+A\overline{B})C &= \overline{A}C+\overline{B}C \quad \text{distributive} \\ (\overline{A}+\overline{B})C &= \overline{A}C+\overline{B}C \quad \text{absorption} \\ \overline{A}C+\overline{B}C &= \overline{A}C+\overline{B}C \quad \text{distributive} \end{aligned}$$

2.6 Answers

47.
$$\begin{aligned}
\overline{AB}(A+B) &= \overline{A}B + A\overline{B} & \\
(\overline{A}+\overline{B})(A+B) &= \overline{A}B + A\overline{B} & \text{De Morgan's} \\
\overline{A}A + \overline{A}B + \overline{B}A + \overline{B}B &= \overline{A}B + A\overline{B} & \text{distributive} \\
0 + \overline{A}B + \overline{B}A + 0 &= \overline{A}B + A\overline{B} & \text{complement} \\
\overline{A}B + A\overline{B} &= \overline{A}B + A\overline{B} & \text{identity}
\end{aligned}$$

48.
$$\begin{aligned}
\overline{\overline{\overline{A}BC} + D} &= \overline{A}BC\overline{D} & \\
\overline{\overline{\overline{A}BC}}\;\overline{D} &= \overline{A}BC\overline{D} & \text{De Morgan's} \\
\overline{A}BC\overline{D} &= \overline{A}BC\overline{D} & \text{complement}
\end{aligned}$$

49.
$$\begin{aligned}
A\overline{B}\;\overline{\overline{A}\;\overline{C}} &= A\overline{B} & \\
A\overline{B}(A+C) &= A\overline{B} & \text{De Morgan's} \\
A\overline{B}A + A\overline{B}C &= A\overline{B} & \text{distributive} \\
A\overline{B} + A\overline{B}C &= A\overline{B} & \text{idempotent} \\
A\overline{B} &= A\overline{B} & \text{absorption}
\end{aligned}$$

Exercises for Section 2.7

In the following exercises, let p denote "The movie was popular" and q denote "The movie will make a lot of money." Translate the following propositions into English sentences.

1. $p \to q$
2. $\sim p \to \sim q$
3. $\sim q \to \sim p$
4. $q \to p$
5. $\sim p \vee q$
6. $p \wedge \sim q$

In the following exercises, let p denote "Pat eats a burger for dinner" and q denote "Pat is too full for dessert." Translate the following sentences into logical symbols.

7. If Pat eats a burger for dinner, she will be too full for dessert.
8. If Pat does not eat a burger for dinner, she will not be too full for dessert.
9. If Pat is too full for dessert, then she ate a burger for dinner.
10. If Pat is not too full for dessert, then she did not eat a burger for dinner.
11. If Pat is too full for dessert, then she did not eat a burger for dinner.
12. Pat being too full for dessert implies that she ate a burger for dinner.
13. Pat not being too full for dessert implies that she did not eat a burger for dinner.
14. Pat not eating a burger for dinner implies that she will not be too full for dessert.
15. Pat eating a burger for dinner implies that she will be too full for dessert.
16. Either Pat does not eat a burger for dinner or she will be too full for dessert.
17. Either Pat is not too full for dessert or she ate a burger for dinner.

2.7 Exercises

18. Either Pat is too full for dessert or she did not eat a burger for dinner.

19. The following conditional statement is true: If Pat is eaten by bears, she will not finish her marking. Given that, answer the following questions.

    (a) Pat is eaten by bears. Did she finish her marking?

    (b) Pat is not eaten by bears. Did she finish her marking?

    (c) Pat finished her marking. Was she eaten by bears?

    (d) Pat did not finish her marking. Was she eaten by bears?

20. The following conditional statement is true: If Rich is asleep, then he is not playing ping-pong. Given that, answer the following questions.

    (a) Rich is playing ping-pong. Is he asleep?

    (b) Rich is asleep. Is he playing ping-pong?

    (c) Rich is not asleep. Is he playing ping-pong?

    (d) Rich is not playing ping-pong. Is he asleep?

Of course, for the previous questions, I chose situations in which you can use common sense to determine the answer. However, the true test of whether you understand the concept is to replace the above propositions by complete nonsense.

21. The following conditional statement is true: If ettercaps are green, then toves are slithy. Given that, answer the following questions.

    (a) Toves are slithy. Are ettercaps green?

    (b) Toves are not slithy. Are ettercaps green?

    (c) Ettercaps are green. Are toves slithy?

    (d) Ettercaps are red. Are toves slithy?

22. The following conditional statement is true: If the hare reads the Times Colonist, the tortoise will take out the recycling. Given that, answer the following questions.

    (a) The hare does not read the Times Colonist. Will the tortoise take out the recycling?

2.7 Exercises

(b) The hare reads the Times Colonist. Will the tortoise take out the recycling?

(c) The tortoise takes out the recycling. Does the hare read the Times Colonist?

(d) The tortoise is not taking out the recycling. Does the hare read the Times Colonist?

Given the conditional statement, "If frattling is non-responsive, then the runges must be strunking", write the corresponding English sentences for the following.

23. The contrapositive ($\sim q \to \sim p$)

24. The converse ($q \to p$)

25. The inverse ($\sim p \to \sim q$)

26. The "or" form ($\sim p \vee q$)

27. Given the conditional statement, "If Bossy is mooing, she must be a cow," which of the four following statements is the contrapositive ($\sim q \to \sim p$)?

    (a) If Bossy is not a cow, she is not mooing.

    (b) If Bossy is a cow, then she is mooing.

    (c) If Bossy is mooing, then she must be a cow.

    (d) If Bossy is not mooing, then she must not be a cow.

28. Given the conditional statement, "If Bossy is mooing, she must be a cow," which of the four following statements is the converse ($q \to p$)?

    (a) If Bossy is not a cow, she is not mooing.

    (b) If Bossy is a cow, then she is mooing.

    (c) If Bossy is mooing, then she must be a cow.

    (d) If Bossy is not mooing, then she must not be a cow.

29. If the statement "If Bossy is mooing, then she must be a cow," is a true statement, which of the four following statements is also true?

    (a) If Bossy is not a cow, she is not mooing.

    (b) If Bossy is a cow, then she is mooing.

2.7 Exercises

(c) Either Bossy is mooing or she is a cow.

(d) If Bossy is not mooing, then she must not be a cow.

30. Which of the following is the correct "or" form for the conditional "If Bossy is mooing, then she must be a cow"?

    (a) Bossy is a cow or she is not mooing.

    (b) Bossy is not a cow or she is not mooing.

    (c) Bossy is not a cow or she is mooing.

    (d) Bossy is a cow or she is mooing.

31. If the statement "If Bossy is mooing, then she must be a cow" is a true statement, which of the following cannot occur?

    (a) Bossy is mooing and she is a cow.

    (b) Bossy is mooing and she is not a cow.

    (c) Bossy is not mooing and she is not a cow.

    (d) Bossy is not mooing and she is a cow.

32. Consider the following "or" form statement, "Either Superman has a cape or he cannot fly." Which of the following is the correct form of the corresponding conditional?

    (a) If Superman does not have a cape, then he cannot fly.

    (b) If Superman has a cape, then he can fly.

    (c) If Superman can fly, then he has a cape.

    (d) If Superman cannot fly, then he doesn't have a cape.

33. Consider the conditional "If John has the flu or misses the bus, he will be late for work". Which of the following is the corresponding contrapositive statement ($\sim q \to \sim p$)?

    (a) If John is late for work, then he had the flu or missed the bus.

    (b) If John is late for work, then he did not have the flu or did not miss the bus.

    (c) If John is not late for work, then he did not have the flu or did not miss the bus.

2.7 Exercises

(d) If John is not late for work, then he did not have the flu and did not miss the bus.

34. Consider the conditional "If Rich doesn't show his work or makes a mistake, then he will not get full credit". Which of the following is the corresponding contrapositive statement $(\sim q \to \sim p)$?

    (a) If Rich received full credit, then he showed his work and did not make a mistake.

    (b) If Rich received full credit, then he showed his work or did not make a mistake.

    (c) If Rich did not get full credit, then he didn't show his work and made a mistake.

    (d) If Rich did not get full credit, then he didn't show his work or made a mistake.

35. Consider the conditional "If Pat is late and has not called her husband, he will be worried". Which of the following is the corresponding contrapositive statement $(\sim q \to \sim p)$?

    (a) If Pat's husband is not worried, then she is not late and did call him.

    (b) If Pat's husband is not worried, then she is not late or did call him.

    (c) If Pat's husband is worried, then she is late and has not called him.

    (d) If Pat's husband is not worried, then she is late and did not call him.

36. Consider the conditional "If grunkles are circular, then runges are square and triptrops are blue". Which of the following is the corresponding contrapositive statement $(\sim q \to \sim p)$?

    (a) If runges are not square and triptrops are not blue, then grunkles are not circular.

    (b) If runges are not square or triptrops are not blue, then grunkles are circular.

    (c) If runges are not square or triptrops are not blue, then grunkles are not circular.

2.7 Exercises

(d) If runges are not square and triptrops are not blue, then grunkles are circular.

Answers to Section 2.7 Exercises

1. If the movie was popular, then it will make a lot of money. (Or: The movie's popularity implies that it will make a lot of money.)
2. If the movie was not popular, then it will not make a lot of money.
3. If the movie did not make a lot of money, then the movie was not popular.
4. If the movie will make a lot of money, then it is popular.
5. The movie was not popular or it made a lot of money.
6. The movie was popular and it did not make a lot of money.
7. $p \to q$
8. $\sim p \to \sim q$
9. $q \to p$
10. $\sim q \to \sim p$
11. $q \to \sim p$
12. $q \to p$
13. $\sim q \to \sim p$
14. $\sim p \to \sim q$
15. $p \to q$
16. $\sim p \vee q$
17. $\sim q \vee p$
18. $q \vee \sim p$
19. a) No b) Maybe c) No d) Maybe
20. a) No b) No c) Maybe d) Maybe
21. a) Maybe b) No c) Yes d) Maybe
22. a) Maybe b) Yes c) Maybe d) No
23. If the runges are not strunking, then the frattling must be responsive.
24. If the runges are strunking, then the frattling is non-responsive.

2.7 Answers

25. If the frattling is responsive, then the runges must not be strunking.
26. The frattling is responsive or the runges are strunking.
27. (a)
28. (b)
29. (a)
30. (a)
31. (b)
32. If you let $p =$ "Superman does not have a cape", then the answer is (a). If instead you let $p =$ "Superman cannot fly", then the answer is (c). So either (a) or (c) would be correct.
33. (d)
34. (a)
35. (b)
36. (c)

Exercises for Section 2.8

Write out the truth tables for the following logical expressions. (You might want to do it as just one or two really big tables.)

1. $p \to q$
2. $\sim p \to \sim q$
3. $\sim q \to \sim p$
4. $q \to p$
5. $\sim p \lor q$
6. $p \land \sim q$
7. $p \leftrightarrow q$
8. $\sim p \leftrightarrow \sim q$
9. $p \oplus q$
10. $p \lor \sim q$
11. $\sim p \oplus \sim q$
12. $(p \to q) \land (q \to p)$
13. $(p \to q) \lor (q \to p)$
14. $(p \to q) \land (\sim p \to \sim q)$
15. $(p \to q) \lor (\sim p \to \sim q)$
16. Looking at your results questions 1-15, which expressions are logically equivalent to $p \leftrightarrow q$?
17. Looking at your results for questions 1-15, which expressions are logically equivalent to $p \to q$?
18. Looking at your results for questions 1-15, which expressions are logically equivalent to $q \to p$?

Consider the following conditional statements. I hope you agree that they all make a certain amount of sense. However, if they were rewritten as **biconditional** statements, would they continue to make sense? Answer True or False.

19. If Barney is a dog, then he has four legs.

2.8 Exercises

20. If Rich is asleep, then he is not playing ping-pong.
21. If Alycia gets 90% or better as her final mark, she will get an A+.
22. If Bossy is mooing, then she is a cow.
23. If Pat sleeps in, she is late for class.
24. If Frank does not pay his bill on time, he will be charged a late charge.
25. If Susan bought her computer less than a year ago, her warranty is still in effect.
26. If Raymond eats a burger for dinner, he will be too full for dessert.

In the following exercises, let p denote "Pat eats a burger for dinner" and let q denote "Pat is too full for dessert." Translate the following sentences into logical symbols.

27. If and only if Pat eats a burger for dinner, she will be too full for dessert.
28. Pat will not be too full for dessert if and only if she did not eat a burger for dinner.
29. If Pat eats a burger for dinner, then she will be too full for dessert.
30. If Pat is not too full for dessert, then she did not eat a burger for dinner.

Are the following two sentences biconditional statements? (In other words, could you replace them by an equivalent "if and only if" construction?)

31. If Frank does not pay his bill on time, then he will be charged a late charge, and if he does pay his bill on time, he will not be charged a late charge.
32. If Alycia gets 90% or better as her final mark, she will get an A+, and if she gets an A+, then she got 90% or better as her final mark.
33. The following conditional statement is true: If and only if Pat is eaten by bears, she will not finish her marking. Given that, answer the following questions.
    (a) Pat is eaten by bears. Did she finish her marking?
    (b) Pat is not eaten by bears. Did she finish her marking?
    (c) Pat finished her marking. Was she eaten by bears?

2.8 Exercises

(d) Pat did not finish her marking. Was she eaten by bears?

34. The following conditional statement is true: If Rich is asleep, then he is not playing ping-pong and vice versa. Given that, answer the following questions.

    (a) Rich is playing ping-pong. Is he asleep?

    (b) Rich is asleep. Is he playing ping-pong?

    (c) Rich is not asleep. Is he playing ping-pong?

    (d) Rich is not playing ping-pong. Is he asleep?

35. The following conditional statement is true: Ettercaps are green if and only if toves are slithy. Given that, answer the following questions.

    (a) Toves are slithy. Are ettercaps green?

    (b) Toves are not slithy. Are ettercaps green?

    (c) Ettercaps are green. Are toves slithy?

    (d) Ettercaps are red. Are toves slithy?

36. If the statement "If and only if Superman has a cape, then he can fly" is a true statement, which of the following cannot occur? (You may choose more than one.)

    (a) Superman has a cape and he can fly.

    (b) Superman has a cape and he cannot fly.

    (c) Superman does not have a cape and cannot fly.

    (d) Superman does not have a cape and can fly.

Answers to Section 2.8 Exercises

Here are the truth tables for the expressions in questions 1 through 15.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	$\sim q \to \sim p$	$q \to p$	$\sim p \vee q$	$p \wedge \sim q$
0	0	1	1	1	1	1	1	1	0
0	1	1	0	1	0	1	0	1	0
1	0	0	1	0	1	0	1	0	1
1	1	0	0	1	1	1	1	1	0

p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim p \leftrightarrow \sim q$	$p \oplus q$	$p \vee \sim q$	$\sim p \oplus \sim q$
0	0	1	1	1	1	0	1	0
0	1	1	0	0	0	1	0	1
1	0	0	1	0	0	1	1	1
1	1	0	0	1	1	0	1	0

p	q	$(p \to q) \wedge (q \to p)$	$(p \to q) \vee (q \to p)$	$(p \to q) \wedge (\sim p \to \sim q)$	$(p \to q) \vee (\sim p \to \sim q)$
0	0	1	1	1	1
0	1	0	1	0	1
1	0	0	1	0	1
1	1	1	1	1	1

16. By comparing columns, the following expressions are logically equivalent to $p \leftrightarrow q$:
    (a) $\sim p \leftrightarrow \sim q$
    (b) $(p \to q) \wedge (q \to p)$
    (c) $(p \to q) \wedge (\sim p \to \sim q)$ (and you may or may not have noticed that it's also equal to $\sim p \oplus q$, which is kind of cool)
17. By comparing columns, the following expressions are logically equivalent to $p \to q$:
    (a) $\sim q \to \sim p$
    (b) $\sim p \vee q$
18. By comparing columns, the following expressions are logically equivalent to $q \to p$:

2.8 Answers

(a) $\sim p \to \sim q$

(b) $p \vee \sim q$

19. False
20. False
21. True
22. False
23. False
24. True
25. True
26. False
27. $p \leftrightarrow q$
28. $\sim q \leftrightarrow \sim p$
29. $p \to q$
30. $\sim q \to \sim p$
31. Yes
32. Yes
33. a) No b) Yes c) No d) Yes
34. a) No b) No c) Yes d) Yes
35. a) Yes b) No c) Yes d) No
36. b) and d)

Mixed Practice

1. Draw the gate diagram that corresponds to the Boolean expression $\overline{A} + \overline{B\,\overline{C}}$. Do not simplify!

2. Use a truth table to simplify the logical expression $(\sim p \wedge \sim q) \oplus (\sim p \wedge q)$.

3. Consider the statement, "This apple is red." Which of the following are logically equivalent to that statement? Circle any correct answers. You may choose more than one.

   (a) It is not true that this apple is not red.

   (b) This apple is red and this apple is red.

   (c) This apple is red or this apple is not red.

   (d) This apple is both red and shiny or this apple is red but not shiny.

   (e) This apple is red or this apple is both red and shiny.

   (f) This apple is red or this apple is not red but it is shiny.

4. Prove the following using the laws of logic. If you're stuck, try using a truth table for part marks.

$$A + (\overline{C} + 0)(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$$

5. The following statement is true: "If you eat at Joe's, then you will have a good meal." Given that, can the following situations occur? Answer "Yes" or "No".

   (a) You did not eat at Joe's and you had a good meal.

   (b) You did not eat at Joe's and you had a bad meal.

   (c) You ate at Joe's and you had a bad meal.

6. Consider the statement $p \to q$: "If you break a mirror, then you will have seven years of bad luck." Which of the following statements are logically equivalent to $p \to q$? Circle all of the correct answers.

   (a) If you don't break a mirror, you won't have seven years of bad luck.

   (b) If you do not have seven years of bad luck, then you did not break a mirror.

Ch2 Mixed Practice

(c) If you have seven years of bad luck, then you broke a mirror.

(d) Either you did not break a mirror or you had seven years of bad luck or both.

7. Consider the statement: "If and only if a quantity is conserved, then a symmetry is exhibited." Answer the following questions with "Yes", "No", or "Maybe".

   (a) A quantity is not conserved. Is a symmetry exhibited?

   (b) A symmetry is exhibited. Is a quantity conserved?

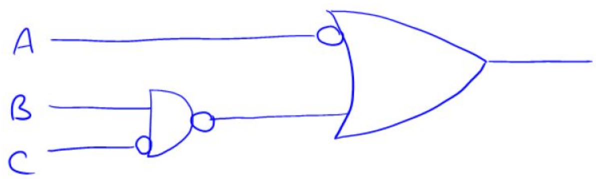
   (c) A symmetry is not exhibited. Is a quantity conserved?

8. Use a truth table to simplify the logical expression $(p \leftrightarrow \sim q) \wedge (p \leftrightarrow q)$.

Ch2 Mixed Practice Answers

Answers

1. Gate diagram for $\overline{A} + \overline{\overline{B}\,\overline{C}}$:



2. Here's the truth table:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim p \wedge q$	$(\sim p \wedge \sim q) \oplus (\sim p \wedge q)$
0	0	1	1	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

The third and seventh columns are the same, so the expression simplifies to $\sim p$

3. (a), (b), (d), and (e) are correct.

Here is the reasoning. Let $p =$"This apple is red" and $q =$"This apple is shiny".

(a) The sentence translates to the expression $\sim(\sim p)$, which is p by complement.

(b) $p \wedge p$, which is p by idempotent.

(c) $p \vee \sim p$, which is 1 by identity and does not equal p

(d) $(p \wedge q) \vee (p \wedge \sim q) \Leftrightarrow p \wedge (q \vee \sim q)$ distributive

$\Leftrightarrow p \wedge 1$ complement

$\Leftrightarrow p$ identity

(e) $p \vee (p \wedge q) \Leftrightarrow p$ absorption

(f) $p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$ absorption, which does not equal p

4. $A + (\overline{C} + 0)(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ (this is the original statement)

$A + (\overline{C})(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

Ch 2 Mixed Practice Answers

$A + (\overline{C})(1) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ complement

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A}}$ idempotent

$A + \overline{C} = \overline{A}\,\overline{C} + A$ complement

$A + \overline{C} = \overline{C} + A$ absorption (you can stop here if you wish)

$A + \overline{C} = A + \overline{C}$ commutative (you can skip this step)

Please note that many different solutions are possible!

5. If $p \to q$ is true, you cannot have p true and q false, so the answers are (a) Yes, (b) Yes, and (c) No.
6. (b) is the contrapositive, and (d) is the "or" form, so both (b) and (d) are correct.
7. If the biconditional $p \leftrightarrow q$ is true, then p and q are either both true or both false. So the answers are: (a) No, (b) Yes, (c) No.
8. Here's the truth table:

p	q	$\sim q$	$p \leftrightarrow \sim q$	$p \leftrightarrow q$	$(p \leftrightarrow \sim q) \wedge (p \leftrightarrow q)$
0	0	1	0	1	0
0	1	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	0

So the expression simplifies to 0.

Exercises for Section 3.1

Predict the next three terms of the following sequences.

1. $18, 16, 14, \ldots$
2. $1, 4, 9, 16, \ldots$
3. $12, 24, 48, 96, \ldots$
4. $144, 36, 9, \ldots$
5. $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, ...$
6. $5, -10, 20, \ldots$
7. $13, 25, 37, 49, \ldots$
8. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$

Give a formula for the general term (the n^{th} term a_n in terms of n) of the following sequences. Use $n = 1$ as your starting index.

9. $1, 4, 9, 16, \ldots$
10. $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, ...$
11. $2, 4, 6, 8, \ldots$
12. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$

Find the first four terms of the following recursively defined sequences.

13. $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 5 \quad \text{for } n \geq 2 \end{cases}$

14. $\begin{cases} a_1 = 10 \\ a_n = 3a_{n-1} \quad \text{for } n \geq 2 \end{cases}$

15. $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_n = a_{n-1} \times a_{n-2} \quad \text{for } n \geq 3 \end{cases}$

16. $\begin{cases} a_1 = 2 \\ a_n = \frac{1}{a_{n-1}} + 1 \quad \text{for } n \geq 2 \end{cases}$

3.1 Exercises

In each of the following, the general formula for the n^{th} term of a sequence is given. Find the first four terms.

17. $a_n = 3n - 5$ for $n \geq 1$
18. $a_n = 3^{n-2}$ for $n \geq 1$
19. $a_n = n!$ for $n \geq 1$
20. $a_n = \frac{1}{n^2}$ for $n \geq 1$

In each of the following, the general formula for the nth term of a sequence is given. Calculate the specified terms.

21. Find a_7 for the sequence $a_n = 5\left(2^{n+1}\right)$ for $n \geq 1$
22. Find a_{100} for the sequence $a_n = 4n + 15$ for $n \geq 1$
23. Find a_{2500} for the sequence $a_n = \frac{n+2}{n+1}$ for $n \geq 1$
24. Find a_{10} for the sequence $a_n = 2n^3$ for $n \geq 1$

Calculate S_3 and S_6 for the following series.

25. $3 + 6 + 9 + \ldots$
26. $1 + 4 + 9 + 16 + \ldots$
27. $5 - 10 + 20 - 40 + \ldots$
28. $5 + 3 + 1 + \ldots$

Write out each sum in full and then evaluate.

29. $\sum_{n=3}^{7} n$
30. $\sum_{j=4}^{10} (-1)^j$
31. $\sum_{i=0}^{4} 2^i$
32. $\sum_{k=20}^{25} (3k - 10)$

Write each series in sigma notation. (Answers may vary.)

33. $1 + 8 + 27 + 64 + \ldots 1000$

3.1 Exercises

34. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+...$

35. $2+4+6+8+\ldots$

36. $2+4+6+8$

Evil alert! The following questions are just for those wanting a challenge. This type of question will not be tested.

37. (nasty) Write the sequence $1, 4, 9, 16, \ldots$ using a **recursive** definition.

38. (thorny) Write the sequence $1, 2, 6, 24, \ldots$ using a **general** formula.

39. (tricksy) Consider the following sequence:

$$4, 5, 20, 100, 2000$$

   (a) What's the next term in this sequence?

   (b) What's the recursive formula for this sequence?

Answers to Section 3.1 Exercises

1. $12, 10, 8$ (pattern is to subtract 2)
2. $25, 36, 49$ (n^{th} term is equal to n^2
3. $192, 384, 768$ (multiply by 2)
4. $\frac{9}{4}, \frac{9}{16}, \frac{9}{64}$ (divide by 4)
5. $\sqrt{7}, 2\sqrt{2}, 3$ (n^{th} term is $\sqrt{n}$)
6. $-40, 80, -160$ (multiply by -2)
7. $61, 73, 85$ (add 12)
8. $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$
9. $a_n = n^2$
10. $a_n = \sqrt{n}$
11. $a_n = 2n$
12. $a_n = \frac{1}{n+1}$
13. $2, 7, 12, 17$
14. $10, 30, 90, 270$
15. $2, 3, 6, 18$
16. $2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$
17. $-2, 1, 4, 7$
18. $\frac{1}{3}, 1, 3, 9$
19. $1, 2, 6, 24$
20. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$
21. $a_7 = 1280$
22. $a_{100} = 415$
23. $a_{2500} = \frac{2502}{2501}$
24. $a_{10} = 2000$
25. $S_3 = 18, S_6 = 63$

3.1 Answers

26. $S_3 = 14, S_6 = 91$

27. $S_3 = 15, S_6 = -105$

28. $S_3 = 9, S_6 = 0$

29. $\sum_{n=3}^{7} n = 3 + 4 + 5 + 6 + 7 = 25$

30. $\sum_{j=4}^{10} (-1)^j = 1 + (-1) + 1 + (-1) + 1 + (-1) + 1 = 1$

31. $\sum_{i=0}^{4} 2^i = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 1 + 2 + 4 + 8 + 16 = 31$

32. $\sum_{k=20}^{25} 3k - 10 = 50 + 53 + 56 + 59 + 62 + 65 = 345$

33. $\sum_{i=1}^{10} i^3$

34. $\sum_{j=2}^{\infty} \frac{1}{j}$

35. $\sum_{k=1}^{\infty} 2k$

36. $\sum_{k=1}^{4} 2k$

37. You could either do

$$\begin{cases} a_1 = 1 \\ a_n = \left(\sqrt{a_{n-1}} + 1\right)^2 & \text{for } n \geq 2 \end{cases}$$

or another possibility is

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 2n - 1 & \text{for } n \geq 2 \end{cases}$$

38. $a_n = n!$ for $n \geq 1$

39. The next term is 200,000.

3.1 Answers

$$\begin{cases} a_1 = 4 \\ a_2 = 5 \\ a_n = a_{n-1} \times a_{n-2} \end{cases}$$

Exercises for Section 3.2

State whether the following sequences are arithmetic or not. If they are, state the first term and common difference.

1. $8, 9, 11, 13, 16, \ldots$
2. $-3, -10, -17, -24, \ldots$
3. $3, 6, 12, 24, \ldots$
4. $1, 2, 6, 24, \ldots$
5. $81, 72, 63, 54, \ldots$
6. $1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, ...$

Give both the general formula and the recursive formula for the n^{th} term a_n of the following arithmetic sequences. Assume that the first term of the sequence is a_1. For the general formula, be sure to simplify your answer.

7. $1, 3, 5, 7, \ldots$
8. $5, -6, -17, -28, \ldots$
9. $-40, -37, -34, -31, \ldots$
10. $24, 28, 32, 36, \ldots$

For the following arithmetic sequences, calculate a_{50} and a_{261}, assuming that the first term is a_1.

11. $18, 16, 14, 12, \ldots$
12. $12, 12.3, 12.6, 12.9, \ldots$

State whether the following recursively defined sequences are arithmetic or not. (Is there an easy way to tell?)

13. $\begin{cases} a_0 = 5 \\ a_n = a_{n-1} + 4 \quad \text{for } n \geq 1 \end{cases}$

14. $\begin{cases} a_1 = 12 \\ a_n = 2a_{n-1} \quad \text{for } n \geq 2 \end{cases}$

15. $\begin{cases} a_1 = 75 \\ a_n = a_{n-1} - 20 \quad \text{for } n \geq 2 \end{cases}$

3.2 Exercises

16. $\begin{cases} a_0 = 6 \\ a_n = a_{n-1} + 1 \quad \text{for } n \geq 1 \end{cases}$

17. $\begin{cases} a_0 = 7 \\ a_n = 2 - a_{n-1} \quad \text{for } n \geq 1 \end{cases}$

18. $\begin{cases} a_1 = 3 \\ a_n = (a_{n-1})^2 \quad \text{for } n \geq 2 \end{cases}$

19. For the following sequence, calculate the 201^{st} term: $5, 15, 25, 35, \ldots$

20. For the following sequence, which term equals 137? $1, 9, 17, 25, \ldots$

21. What is the common difference for the arithmetic sequence with $a_1 = 200$ and $a_{12} = -240$?

22. Calculate the first term for the arithmetic sequence with common difference 7 whose sixteenth term is 102.

23. Calculate the first four terms of the arithmetic sequence in which the sixth term is 17 and the sixtieth term is 179.

24. Calculate the first four terms of the arithmetic sequence in which the one hundredth term is 403 and the sixty-fourth term is 259.

25. Give a general formula for the arithmetic sequence in which the twentieth term is -107 and the thirty-fifth term is -152.

26. Give a recursive formula for the arithmetic sequence in which the eleventh term is 44 and the fifty-second term is 290.

27. Calculate S_{20} for the series $100 + 97 + 94 + \ldots$

28. Evaluate the series $12 + 17 + 22 + \ldots 82$.

29. Evaluate the series $144 + 138 + 132 + \ldots 78$.

30. Calculate S_{100} for the series $-20 - 16 - 12 - \ldots$

31. Calculate the sum of the odd numbers between 100 and 500.

32. Find the sum of the integers from 50 to 125, inclusive.

Calculate the following sums.

33. $\sum_{k=0}^{53} (5k - 1)$

3.2 Exercises

34. $\sum_{j=10}^{92} 6j$

35. $\sum_{i=30}^{140} (2i+7)$

36. $\sum_{k=3}^{502} (17-3k)$

37. In a supermarket display, there are 37 cans in the bottom layer, 35 in the next layer up, 33 in the next, and so on. How many layers are there if there are 7 cans in the top row?

38. In the previous problem, how many cans are there altogether?

39. In an old-fashioned theatre, there are 25 seats in the first row, 26 in the next, 27 in the one after, and so on. If there are 20 rows in total, how many seats are there altogether?

Answers to Section 3.2 Exercises

1. not arithmetic
2. yes, $d = -7$
3. no
4. no
5. yes, $d = -9$
6. yes, $d = \frac{1}{4}$
7. $a_n = 2n - 1$ and $\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 2 \end{cases}$
8. $a_n = 16 - 11n$ and $\begin{cases} a_1 = 5 \\ a_n = a_{n-1} - 11 \end{cases}$
9. $a_n = 3n - 43$ and $\begin{cases} a_1 = -40 \\ a_n = a_{n-1} + 3 \end{cases}$
10. $a_n = 4n + 20$ and $\begin{cases} a_1 = 24 \\ a_n = a_{n-1} + 4 \end{cases}$
11. $a_n = 20 - 2n$, so $a_{50} = -80$ and $a_{261} = -502$
12. $a_n = 11.7 + 0.3n$, so $a_{50} = 26.7$ and $a_{261} = 90$
13. first four terms are $5, 9, 13, 17$, so arithmetic with $d = 4$
14. first four terms are $12, 24, 48, 96$, so not arithmetic
15. first four terms are $75, 55, 35, 15$, so arithmetic with $d = -20$
16. first four terms are $6, 7, 8, 9$, so arithmetic with $d = 1$
17. first four terms are $7, -5, 7, -5$, so not arithmetic
18. first four terms are $3, 9, 81, 6561$, so not arithmetic
19. $a_n = 10n - 5$, so $a_{201} = 2005$
20. $a_n = 8n - 7$, so $n = 18$
21. $d = -40$

3.2 Answers

22. $a_1 = -3$
23. $a_1 = 2$ and $d = 3$, so the first four terms are $2, 5, 8, 11$
24. $a_1 = 7$ and $d = 4$, so the first four terms are $7, 11, 15, 19$
25. $a_n = -3n - 47$
26. $\begin{cases} a_1 = -16 \\ a_n = a_{n-1} + 6 \end{cases}$
27. $S_{20} = 1430$
28. $S_{15} = 705$
29. $S_{12} = 1332$
30. $S_{100} = 17800$
31. $S_{200} = 60000$
32. $S_{76} = 6650$
33. $S_{53} = 7101$
34. $S_{83} = 25398$
35. $S_{111} = 19647$
36. $S_{500} = -370,250$
37. $n = 16$
38. $S_{16} = 352$
39. $S_{20} = 690$

Exercises for Section 3.3

State whether the following sequences are geometric or not. If they are, state the first term and common ratio.

1. $8, 9, 11, 13, 16, \ldots$
2. $-3, -10, -17, -24, \ldots$
3. $3, 6, 12, 24, \ldots$
4. $1, 2, 6, 24, \ldots$
5. $81, 72, 63, 54, \ldots$
6. $72, 48, 32, ...$

Give both the general formula and the recursive formula for the n^{th} term a_n of the following sequences. Use the convention $n \geq 1$.

7. $1, 3, 9, 27, \ldots$
8. $64, 16, 4, 1, \ldots$
9. $3, -6, 12, -24, \ldots$
10. $24, 2.4, 0.24, \ldots$

For the following sequences, calculate a_{50} and a_{261}, assuming that the first term is a_1.

11. $12, 18, 27, \ldots$
12. $12, 8, \frac{16}{3}, \ldots$

State whether the following recursively defined sequences are geometric or not. (Is there an easy way to tell?)

13. $\begin{cases} a_1 = 5 \\ a_n = a_{n-1} + 4 \quad \text{for } n \geq 2 \end{cases}$

14. $\begin{cases} a_0 = 12 \\ a_n = 2a_{n-1} \quad \text{for } n \geq 1 \end{cases}$

15. $\begin{cases} a_0 = 75 \\ a_n = 10a_{n-1} \quad \text{for } n \geq 1 \end{cases}$

3.3 Exercises

16. $\begin{cases} a_1 = 7 \\ a_n = 2 - a_{n-1} \quad \text{for } n \geq 2 \end{cases}$

17. $\begin{cases} a_1 = 8 \\ a_n = -a_{n-1} \quad \text{for } n \geq 2 \end{cases}$

18. $\begin{cases} a_0 = 3 \\ a_n = (a_{n-1})^2 \quad \text{for } n \geq 1 \end{cases}$

19. For the following sequence, calculate the 201^{st} term: $5, 15, 45, \ldots$

20. For the following sequence, calculate the 20^{th} term: $7, -14, 28, \ldots$

21. Calculate S_{20} for the series $100 + 50 + 25 + \ldots$

22. Calculate S_{20} for the series $100 + 200 + 400 + \ldots$

Calculate the sum, if it exists, for the following series.

23. $-6 + 4 - \frac{8}{3} + ...$

24. $100 + 50 + 25 + \ldots$

25. $100 + 200 + 400 + \ldots$

26. $12 + 3 + \frac{3}{4} + \ldots$

Calculate the following sums, if they exist.

27. $\sum_{k=0}^{10} 2^{k+2}$

28. $\sum_{j=1}^{\infty} 15\left(\frac{3}{5}\right)^j$

29. $\sum_{i=2}^{\infty} 25(0.1)^i$

30. $\sum_{i=0}^{\infty} 4(-3)^i$

31. If the number of vampires in Transylvania doubles every month, then how many vampires will be in Transylvania in 3 years, starting from one individual? Comment on your result if the total population of Transylvania is 2 million people.

3.3 Exercises

32. As I was going to St. Ives, I met a man with seven wives. Each wife had seven sacks. Each sack had seven cats. Each cat had seven kits. Kits, cats, sacks, wives: does this form a geometric sequence?

33. The paper used in the photocopier by Pat's office is said to be 0.097 mm thick. If it is folded over repeatedly, doubling its thickness each time, how thick will the paper be if it's folded 7 times? Bonus: why, then, were the Mythbusters having so many problems trying to fold the paper this many times?

Answers to Section 3.3 Exercises

1. no
2. no
3. yes, $r = 2$
4. no
5. no
6. yes, $r = \frac{2}{3}$
7. $a_n = (3)^{n-1}$ and $\begin{cases} a_1 = 1 \\ a_n = 3a_{n-1} \end{cases}$
8. $a_n = 64\left(\frac{1}{4}\right)^{n-1}$ and $\begin{cases} a_1 = 64 \\ a_n = \frac{a_{n-1}}{4} \end{cases}$
9. $a_n = 3(-2)^{n-1}$ and $\begin{cases} a_1 = 3 \\ a_n = -2a_{n-1} \end{cases}$
10. $a_n = 24(0.1)^{n-1}$ and $\begin{cases} a_1 = 24 \\ a_n = 0.1 \times a_{n-1} \end{cases}$
11. $a_n = 12\left(\frac{3}{2}\right)^{n-1}$, so $a_{50} \approx 5.1 \times 10^9$ and $a_{261} \approx 7.3 \times 10^{46}$
12. $a_n = 12\left(\frac{2}{3}\right)^{n-1}$, so $a_{50} \approx 2.8 \times 10^{-8}$ and $a_{261} \approx 1.97 \times 10^{-45}$
13. no
14. yes, with $r = 2$
15. yes, with $r = 10$
16. no
17. yes, with $r = -1$
18. no
19. $a_n = 5(3)^{n-1}$, so $a_{201} = 5(3)^{200} = 1.33 \times 10^{96}$
20. $a_n = 7(-2)^{n-1}$, so $a_{20} = 7(-2)^{19} = -3,670,016$

3.3 Answers

21. $S_{20} = 200$ (the exact answer is $\frac{26214375}{131072}$ or 1.99980926513671875, but if you round to three decimals, the answer is 200.000)

22. $S_{20} = 104,857,500$

23. $S_\infty = \frac{a_1}{1-r} = \frac{-6}{1-(-2/3)} = -\frac{18}{5} = -3.6$

24. $S_\infty = 200$

25. S_∞ does not exist ($r > 1$)

26. $S_\infty = 16$

27. $S_{11} = 2^2 + 2^3 + 2^4 + ... + 2^{12} = \frac{a_1(1-r^n)}{1-r} = \frac{2^2(1-2^{11})}{1-2} = 8188$

28. $S_\infty = 22.5$

29. $S_\infty = \frac{5}{18} = 0.2\overline{7}$

30. S_∞ does not exist ($r < -1$)

31. 3 years is 36 months, so we have a 36-term sequence starting with $1, 2, 4, 8, \ldots$ The n^{th} term will be $a_n = 1(2)^{n-1}$, so the 36^{th} term will be $a_{36} = 1(2)^{35} = 34,359,738,368$, which is a tad larger than the total population of Transylvania.

32. 1 man

    7 wives

    #sacks = #wives × #sacks/wife = 7 × 7 = 49

    #cats = #sacks × #cats/sack = 49 × 7 = 343

    #kits = #cats × #kits/cat = 343 × 7 = 2401

    So kits, cats, sacks, and wives is the sequence $2401, 373, 49, 7$, which is geometric with four terms: $a_1 = 2401$ and $r = \frac{1}{7}$.

33. The paper is initially 0.097 mm thick with no folds. After one fold, the thickness will be 0.097×2, after two folds $0.097 \times 2 \times 2$, etc. So our starting term (zero folds) will be $a_0 = 0.097$ and then will double with r = 2 thereafter, where n is not only the index but also the number of folds made. So $a_n = 0.097(2)^n$, and the term with seven folds will be $a_7 = 0.097(2)^7 = 12.416$, so we can conclude that the paper thickness will be 12.4 mm, or just over 1 cm thick. (The Mythbusters realized that the problems with paperfolding lie with the fold itself, and making

3.3 Answers

the fold lie as flat as possible. If I remember correctly, they resorted to C-clamps and hitting the fold with a hammer to flatten it.)

Ch3

Mixed Practice

1. Label the following sequences as "arithmetic", "geometric" or "neither".
   (a) $1, 1, 2, 3, 5, 8, \ldots$
   (b) $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \ldots$
   (c) $58, 48, 38, \ldots$
2. Consider the sequence given by the following.
   $$a_n = 30 - 3n, \qquad 1 \leq n \leq 3$$
   (a) Is this formula recursive or general?
   (b) Calculate all terms of this sequence.
3. Evaluate the following sum, if it exists. If it doesn't exist, state why not. Show your work!
   $$\sum_{i=2}^{\infty} 8(-3)^i$$
4. Calculate the first three terms of the following sequence.
   $$\begin{cases} a_1 = 3 \\ a_n = (a_{n-1})^2 \quad \text{for} \quad n \geq 2 \end{cases}$$
5. Write a recursive formula for the sequence defined below.
   $$a_n = 7 * 3^n \quad \text{for} \quad n \geq 1$$
6. State whether the following are arithmetic, geometric, or neither. Also, give a formula for the nth term of the sequence. Use a starting index of one.
   (a) $15, 9, 3, -3, \ldots$
   (b) $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
   (c) $48, 12, 3, \frac{3}{4}, \ldots$

Ch 3 Mixed Practice

7. Calculate the following sums, if possible. If not possible, state why not. Show your work.
   (a) $\sum_{j=0}^{4}(3j)$
   (b) $2+4+6+\ldots+88$
   (c) $\sum_{m=0}^{\infty} 300(0.99^m)$
   (d) $\frac{1}{25}-\frac{1}{20}+\frac{1}{16}-\frac{5}{64}+\ldots$
8. Calculate the sum of the odd numbers between 1000 and 5000. Show your work.

Ch 3 Mixed Practice Answers

Answers

1. (a) neither
   (b) geometric
   (c) arithmetic
2. (a) general
   (b) 27, 24, 21
3. undefined, because it is geometric with $r = -3$, and $|r| < 1$ is false
4. 3, 9, 81
5. $\begin{cases} a_1 = 21 \\ a_n = 3a_{n-1} \quad \text{for} \quad n \geq 2 \end{cases}$
6. (a) arithmetic
   either $a_n = 21 - 6n$ (general) or
   $\begin{cases} a_1 = 15 \\ a_n = a_{n-1} - 6 \quad \text{for} \quad n \geq 2 \end{cases}$ (recursive)
   (b) neither
   $a_n = \frac{n-1}{n}$
   (c) geometric,
   either $a_n = 48\left(\frac{1}{4}\right)^{n-1}$ (general) or
   $\begin{cases} a_1 = 48 \\ a_n = \frac{1}{4}a_{n-1} \quad \text{for} \quad n \geq 2 \end{cases}$ (recursive)

Ch 3 Mixed Practice Answers

7. (a) $\sum_{j=0}^{4}(3j) = 0 + 3 + 6 + 9 + 12 = 30$

   (b) arithmetic with $d = 2$ and $n = 44$, so $S_{44} = 1980$

   (c) geometric with $a_m = 300$ and $r = 0.99$, so $S_\infty = 30000$

   (d) geometric with $r = -\frac{5}{4}$, so sum does not exist

8. $1001 + 1003 + 1005 + \ldots + 4999$
   arithmetic series with $d = 2$
   number of terms:
   $a_n = a_m + (n - m)d$
   let's start with $m = 1$
   $4999 = 1001 + (n - 1)2$
   solving for n gives $n = 2000$
   then $S_n = \frac{n}{2}(a_m + a_n)$
   and $S_{2000} = 6\ 000\ 000$

Exercises for Section 4.1

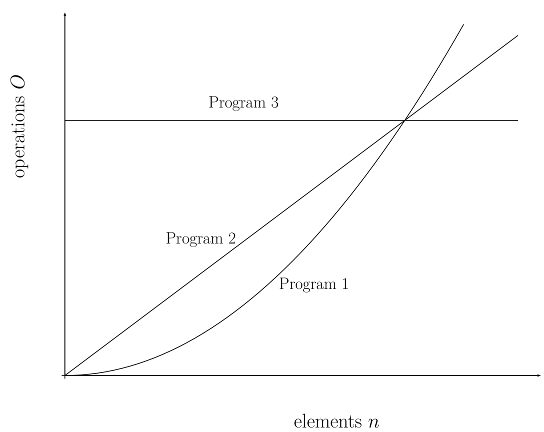
1. Many parts of the world have highways that are toll roads, so that you have to pay a fee to drive on them. For example, before 2008 the Coquihalla Highway in southern BC had a toll for cars of $10 but saved drivers at least an hour in travel time over the alternate route.

   If you were driving in that part of BC, which route should you take (Coquihalla vs. alternate route) if you are

   (a) broke?

   (b) running late and are not broke?

2. This graph shows the number of operations O required to complete a task of size n elements for Programs 1, 2, and 3, where Program 1 is the curved line, Program 2 is the straight line through the origin, and Program 3 is the horizontal line.



   Indicate whether the following statements are true or false.

   (a) There is a certain size of task n where all three programs require the same number of steps.

   (b) Program 2 is a good choice for all sizes of n because it is the "middle ground" between Programs 1 and 3.

   (c) There is no value of n for which Program 2 is clearly the best choice.

4.1 Exercises

(d) For large n, Program 3 will finish faster than the other two programs because the line for Program 3 is below the lines for the other programs on the right-hand side of the graph.

(e) Whether Program 1 is more efficient than Program 3 depends on the size of n.

3. You are living in an apartment block with a single washer and dryer in the basement. You have the choice of doing your laundry one load at a time using the machines downstairs, or you can drive to the laundromat and use many machines at once. Each load of laundry takes one hour to wash and dry using either your apartment's machines or the laundromat's. The laundromat is 30 minutes away by car.

   (a) Under what conditions is your apartment's washer/dryer the fastest way to do your laundry?

   (b) Under what conditions will the two different options take about the same amount of time?

   (c) If you have many loads of clothing, which is the better option?

4. You are playing a computer game and you have a choice of playing a fighter, a cleric, or a wizard. In combat, the fighter always does 50 points worth of damage no matter what level the fighter is. The cleric does 10 points of damage per level, while the damage the wizard does is equal to the square of the level.

   (a) Which character choice (fighter/cleric/wizard) does the most damage at low levels? At high levels?

   (b) At what level is the breakeven point between fighter and cleric?

   (c) At what level does the wizard start to do more damage than the fighter?

Answers to Section 4.1 Exercises

1. (a) alternate route

   (b) Coquihalla

2. (a) true

   (b) false

   (c) true

   (d) true

   (e) true

3. (a) You only have one load.

   (b) You have two loads.

   (c) The laundromat.

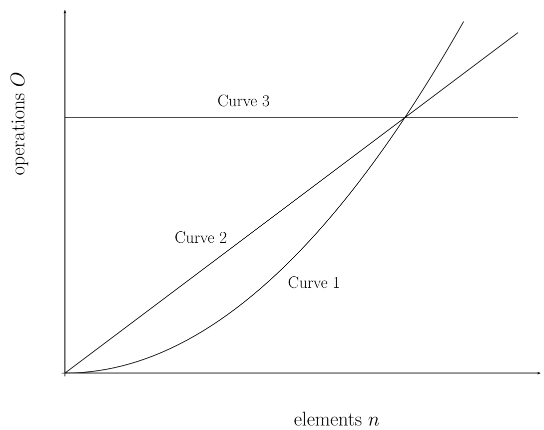
4. (a) Low levels: fighter. High levels: wizard.

   (b) At 5th level, both do the same amount of damage.

   (c) At 8th level.

Exercises for Section 4.2

1. Match the Big O notation with its corresponding curve on the graph.

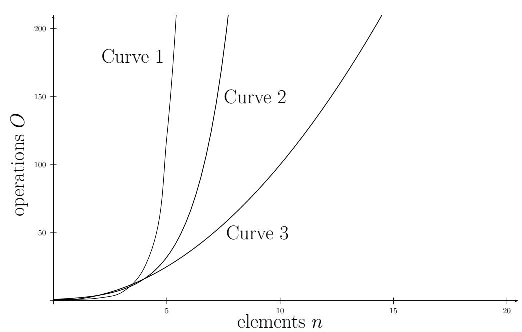


   (a) $O(n^2)$ ____________

   (b) $O(1)$ ____________

   (c) $O(n)$ ____________

2. Match the Big O notation with its corresponding curve on the graph.



   (a) $O(n^2)$ ____________

   (b) $O(n!)$ ____________

   (c) $O(2^n)$ ____________

3. For a task of size n, Program A will always take one million steps to run and Program B will take $5n^2$ steps to run. Indicate whether the

4.2 Exercises

following statements are true or false.

(a) For small n, Program B will run faster than Program A.

(b) For large n, Program B will run faster than Program A.

(c) Program A should always take the same amount of time to run.

(d) If you know the size of the task, you can choose whether Program A or Program B will be more efficient.

(e) If you do not know the size of task, Program B is a good choice because most of the time it will finish faster than Program A.

4. For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

(a) $n^2 + 2n + 3n!$

(b) $7n + 5$

(c) 50

(d) $20n^2 + 40(2^n)$

Answers to Section 4.2 Exercises

1. (a) Curve 1
   (b) Curve 3
   (c) Curve 2
2. (a) Curve 3
   (b) Curve 1
   (c) Curve 2
3. (a) true
   (b) false
   (c) true
   (d) true
   (e) false
4. (a) $O(n!)$
   (b) $O(n)$
   (c) $O(1)$
   (d) $O(2^n)$

Exercises for Section 4.3

1. Suppose you are trying to find an entry in an ordered list. You try two different methods:

   - Method 1: You start at the beginning of the list and go down until you find the entry you want. This has $O(n)$.

   - Method 2: You go to the halfway point and deterine whether the entry of interest is above or below the that middle entry. Then divide that part of the list in half and check the halfway point. Repeat until you've found the entry of interest. This is called a binary search and has $O(\log n)$.

   Answer the following questions about the above scenario.

   (a) If the list has only 10 items and you are not using a computer for this task, then the most efficient method is probably __________.

   (b) If the list is very long, then the most efficient method is definitely __________.

   (c) For method 1, the best case scenario is that the entry you want is located in the following place:

   top / middle / bottom of the list

   (d) For method 1, the worst case scenario is that the entry you want is located in the following place:

   top / middle / bottom of the list

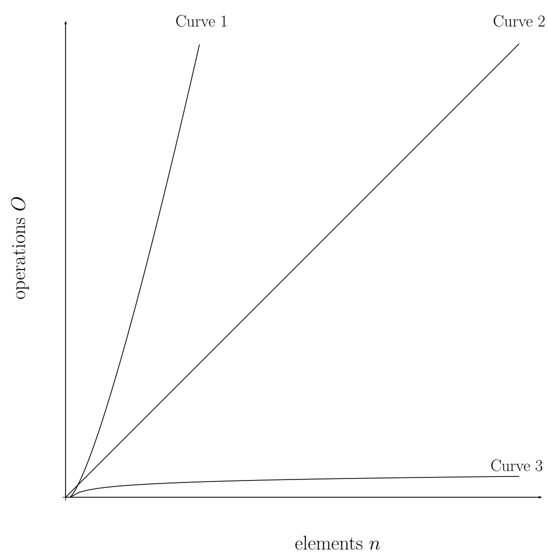
   (e) For method 2, the best case scenario is that the entry you want is located in the following place:

   top / middle / bottom of the list

4.3 Exercises

2. Evaluate the following logarithms. Give exact answers.
   (a) $\log_4(16)$
   (b) $\log_{10}(10^6)$
   (c) $\log_{10}(10)$
   (d) $\log_2(256)$

3. Match the Big O notation with its corresponding curve on the graph. Curve 2 is a straight line.



   (a) $O(\log n)$ __________
   (b) $O(n \log n)$ __________
   (c) $O(n)$ __________

4.3 Exercises

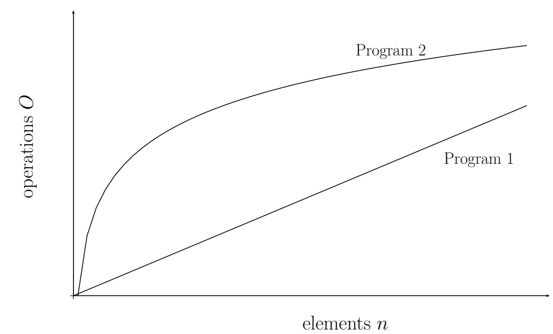
4. Indicate whether the following statements about the $O(\log n)$ curve are true or false.

   (a) If n gets large enough, the curve of $O(\log n)$ will eventually curve downward.

   (b) If n gets large enough, the curve of $O(\log n)$ will reach a certain value and stay there.

   (c) No matter how big n is, the curve of $O(\log n)$ will always increase.

5. This graph shows the number of operations O required to complete a task of size n elements for Programs 1 and 2, where Program 1 is a straight line and Program 2 is a curved line.



   Indicate whether the following statements are true or false.

   (a) Program 1 could be $O(n \log n)$.

   (b) Program 2 could be $O(n \log n)$.

   (c) Program 2 could be $O(\log n)$.

   (d) For large n, Program 1 will finish faster because the line for Program 1 is below the line for Program 2 at the right-hand side of the graph.

6. For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

   (a) $n^2 + 2n \log n + 3 \log n$

   (b) $7n + 9n \log n$

   (c) $7 + 2 \log n$

4.3 Exercises

(d) $\log n + 3n$

(e) $n \log n + 3n!$

(f) $(n + 1) \log n$

Answers to Section 4.3 Exercises

1. (a) Method 1
   (b) Method 2
   (c) top
   (d) bottom
   (e) middle
2. (a) $\log_4(16) = 2$
   (b) $\log_{10}(10^6) = 6$
   (c) $\log_{10}(10) = 1$
   (d) $\log_2(256) = 8$
3. (a) Curve 3
   (b) Curve 1
   (c) Curve 2
4. (a) false
   (b) false
   (c) true
5. (a) false, $O(n \log n)$ has a slight curve to it and the question said that Program 1 is a straight line
   (b) false
   (c) true
   (d) false
6. (a) n^2
   (b) $n \log n$
   (c) $\log n$
   (d) n
   (e) $n!$
   (f) $n \log n$

Ch 4

Mixed Practice

1. You need to buy a lawnmower, and you have researched the following options. You could get a gas mower for around \$250, an electric mower for \$200, or a robot mower for \$1300.

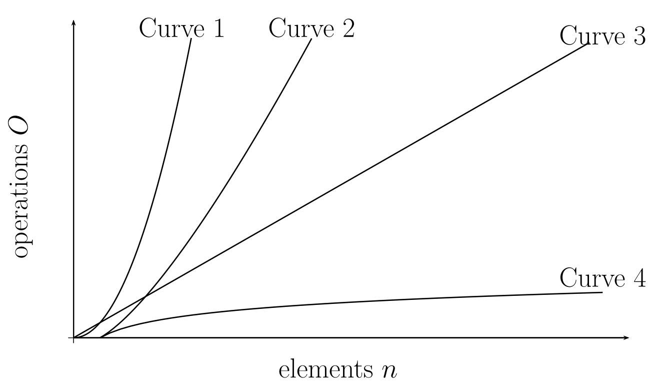
   Indicate whether the following statements are true or false.

   (a) Which mower is the best choice depends on what your priorities are.

   (b) The larger the lawn is, the longer it will take you to mow if you choose either the gas or electric mower options.

   (c) If your lawn is twice as big as your neighbour's, it will take you twice as long as your nieghbour to mow it with either the gas or electric mowers (provided of course that your neighbour is using the same type of mower), so the time to mow would probably be $O(n)$.

2. Match the Big O notation with its corresponding curve on the graph. Please note that curve 3 is a straight line.



   (a) $O(n \log n)$

   (b) $O(\log n)$

   (c) $O(n^2)$

   (d) $O(n)$

3. Evaluate the following logarithms. Give exact answers.

   (a) $\log_3(81)$

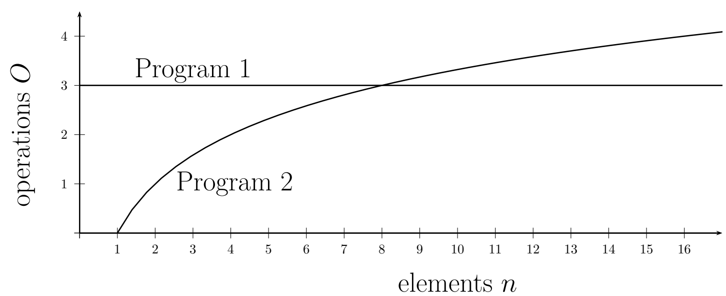
Ch4 Mixed Practice

(b) $\log_{10}(0.01)$

(c) $\log_2(1)$

(d) $\log_4(4)$

4. The following graph shows the number of operations O required to complete a task of size n for Programs 1 and 2. The number of operations required for Program 1 is a constant, so Program 1 is a horizontal straight line.



Indicate whether the following statements are true or false.

(a) It's possible that for a certain value of n, the two programs are equally efficient.

(b) Program 2 is a better choice than Program 1 for some circumstances.

(c) If Program 2 is $O(\log n)$, then for large values of n it could curve downwards and become more efficient than Program 1.

5. If you look up algorithms on how to search a list, you will find that in terms of operations, a linear search has $O(n)$ while a binary search has $O(\log n)$.

Based only on this information, which method is more efficient for large values of n? Indicate the correct choice.

(a) linear seach

(b) binary search

(c) they both have the same efficiency

Why?

Ch 4 Mixed Practice

(a) Because n grows faster than $\log n$ and bigger is better.

(b) There is not enough information to decide.

(c) Because $\log n$ grows slower than n and fewer operations means that the program will run faster.

(d) Because n and $\log n$ grow at the same rate.

6. For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

   (a) $2^n + 5n!$

   (b) $\log n + n$

   (c) $3 + 2 + 1!$

   (d) $n(n + \log n + 1)$

   (e) $n \log n + 2n$

Ch4 Mixed Practice Answers

Answers

1. (a) true
   (b) true
   (c) true
2. (a) Curve 2
   (b) Curve 4
   (c) Curve 1
   (d) Curve 3
3. (a) $\log_3(81) = 4$
   (b) $\log_{10}(0.01) = -2$
   (c) $\log_2(1) = 0$
   (d) $\log_4(4) = 1$
4. (a) true
   (b) true
   (c) false
5. binary search, because $\log n$ grows slower than n and fewer operations means that the program will run faster.
6. (a) $O(n!)$
   (b) $O(n)$
   (c) $O(1)$
   (d) $O(n^2)$
   (e) $O(n \log n)$