Section 7.2

9. $\boxed{\text{Yes}}$

All entries of P are positive.

13. $$P = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.4 & 0.6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.84 \\ 0 & 1 \end{bmatrix}$$

$$P^3 = PP^2$$

$$= \begin{bmatrix} & \\ 0 & 1 \end{bmatrix}\begin{bmatrix} & \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} & \\ 0 & 1 \end{bmatrix}$$

$$\vdots$$

Every power of P will have a 0 in bottom row.

$\boxed{\text{No}}$

15. $$P = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.2 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0.8 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.2 \\ 0.16 & 0.84 \end{bmatrix}$$

All entries of P^2 are positive.

$\boxed{\text{Yes}}$

(23) Let $S = [s_1 \ \ s_2]$

$$SP = S$$

$$[s_1 \ \ s_2] \begin{bmatrix} 0.1 & 0.9 \\ 0.6 & 0.4 \end{bmatrix} = [s_1 \ \ s_2]$$

$$[0.1s_1 + 0.6s_2 \quad 0.9s_1 + 0.4s_2] = [s_1 \quad s_2]$$

$$0.1s_1 + 0.6s_2 = s_1 \Rightarrow -0.9s_1 + 0.6s_2 = 0 \quad (1)$$

$$0.9s_1 + 0.4s_2 = s_2 \Rightarrow 0.9s_1 - 0.6s_2 = 0 \quad (2)$$

$$s_1 + s_2 = 1 \quad (3)$$

$$\begin{array}{c} \begin{array}{ccc} s_1 & s_2 & \# \end{array} \\ \left[\begin{array}{cc|c} 1 & 1 & 1 \\ -0.9 & 0.6 & 0 \\ 0.9 & -0.6 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{l} \\ R_2 + 0.9R_1 \\ R_3 - 0.9R_1 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1.5 & 0.9 \\ 0 & -1.5 & -0.9 \end{array}\right]$$

$$\begin{array}{l} \\ \frac{R_2}{1.5} \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 0.6 \\ 0 & -1.5 & -0.9 \end{array}\right]$$

$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 + 1.5R_2 \end{array} \begin{array}{c} \begin{array}{cc} s_1 & s_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 0.4 \\ 0 & 1 & 0.6 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$$s_1 = 0.4$$

$$s_2 = 0.6$$

$$S = [0.4 \quad 0.6]$$

(25)

Let $S = [s_1 \quad s_2]$

$$SP = S$$

$$[s_1 \quad s_2]\begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix} = [s_1 \quad s_2]$$

$$[0.5s_1 + 0.3s_2 \quad 0.5s_1 + 0.7s_2] = [s_1 \quad s_2]$$

$0.5s_1 + 0.3s_2 = s_1 \Rightarrow -0.5s_1 + 0.3s_2 = 0$ (1)

$0.5s_1 + 0.7s_2 = s_2 \Rightarrow 0.5s_1 - 0.3s_2 = 0$ (2)

$s_1 + s_2 = 1$ (3)

$$\begin{array}{ccc} s_1 & s_2 & \# \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -0.5 & 0.3 & 0 \\ 0.5 & -0.3 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 + 0.5R_1 \\ R_3 - 0.5R_1 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0.8 & 0.5 \\ 0 & -0.8 & -0.5 \end{array}\right]$$

$$\frac{R_2}{0.8} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 0.625 \\ 0 & -0.8 & -0.5 \end{array}\right]$$

$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 + 0.8R_2 \end{array} \begin{array}{c} \begin{array}{cc} s_1 & s_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 0.375 \\ 0 & 1 & 0.625 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$s_1 = 0.375$

$s_2 = 0.625$

$$S = [0.375 \quad 0.625]$$

(27) Let $S = [s_1 \ s_2 \ s_3]$

$$SP = S$$

$$[s_1 \ s_2 \ s_3] \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.7 & 0 \\ 0 & 0.6 & 0.4 \end{bmatrix} = [s_1 \ s_2 \ s_3]$$

$$[0.5s_1 + 0.3s_2 \quad 0.1s_1 + 0.7s_2 + 0.6s_3 \quad 0.4s_1 + 0.4s_3] = [s_1 \ s_2 \ s_3]$$

$$0.5s_1 + 0.3s_2 = s_1 \Rightarrow -0.5s_1 + 0.3s_2 = 0 \quad (1)$$

$$0.1s_1 + 0.7s_2 + 0.6s_3 = s_2 \Rightarrow 0.1s_1 - 0.3s_2 + 0.6s_3 = 0 \quad (2)$$

$$0.4s_1 + 0.4s_3 = s_3 \Rightarrow 0.4s_1 - 0.6s_3 = 0 \quad (3)$$

$$s_1 + s_2 + s_3 = 1 \quad (4)$$

$$\begin{array}{cccc} s_1 & s_2 & s_3 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -0.5 & 0.3 & 0 & 0 \\ 0.1 & -0.3 & 0.6 & 0 \\ 0.4 & 0 & -0.6 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 + 0.5R_1 \\ R_3 - 0.1R_1 \\ R_4 - 0.4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0.8 & 0.5 & 0.5 \\ 0 & -0.4 & 0.5 & -0.1 \\ 0 & -0.4 & -1 & -0.4 \end{array}\right]$$

$$\frac{R_2}{0.8} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0.625 & 0.625 \\ 0 & -0.4 & 0.5 & -0.1 \\ 0 & -0.4 & -1 & -0.4 \end{array}\right]$$

$\rightarrow$

(27) Cont'd

$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 + 0.4R_2 \\ R_4 + 0.4R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0.375 & 0.375 \\ 0 & 1 & 0.625 & 0.625 \\ 0 & 0 & 0.75 & 0.15 \\ 0 & 0 & -0.75 & -0.15 \end{array}\right]$$

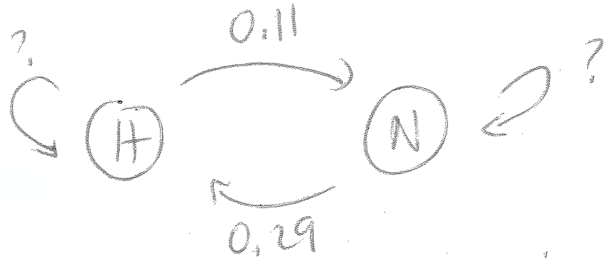
$$\frac{R_3}{0.75} \left[\begin{array}{ccc|c} 1 & 0 & 0.375 & 0.375 \\ 0 & 1 & 0.625 & 0.625 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & -0.75 & -0.15 \end{array}\right]$$

$$\begin{array}{ll} & \begin{array}{cccc} s_1 & s_2 & s_3 & \end{array} \\ \begin{array}{l} R_1 - 0.375R_3 \\ R_2 - 0.625R_3 \\ \\ R_4 + 0.75R_3 \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.3 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

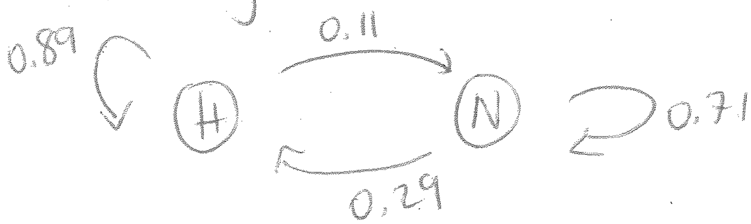
$$s_1 = 0.3$$
$$s_2 = 0.5$$
$$s_3 = 0.2$$

$$S = [0.3 \quad 0.5 \quad 0.2]$$

(51) Let H = home tracks
N = national pool



Arrows leaving a state sum to 1.



$$P = \begin{array}{cc} & \begin{array}{cc} H & N \end{array} \\ \begin{array}{c} H \\ N \end{array} & \begin{bmatrix} 0.89 & 0.11 \\ 0.29 & 0.71 \end{bmatrix} \end{array} \leftarrow \text{next state}$$

↑ current state

In the long run, the state will approach S.

Find S.

Let $S = [s_1 \quad s_2]$

$$SP = S$$

$$[s_1 \quad s_2] \begin{bmatrix} 0.89 & 0.11 \\ 0.29 & 0.71 \end{bmatrix} = [s_1 \quad s_2]$$

$$[0.89 s_1 + 0.29 s_2 \qquad 0.11 s_1 + 0.71 s_2] = [s_1 \quad s_2]$$

→

51 Cont'd

$0.89S_1 + 0.29S_2 = S_1 \Rightarrow -0.11S_1 + 0.29S_2 = 0$ (1)

$0.11S_1 + 0.71S_2 = S_2 \Rightarrow 0.11S_1 - 0.29S_2 = 0$ (2)

$S_1 + S_2 = 1$ (3)

$$\begin{array}{ccc} S_1 & S_2 & \# \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -0.11 & 0.29 & 0 \\ 0.11 & -0.29 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 + 0.11R_1 \\ R_3 - 0.11R_1 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0.4 & 0.11 \\ 0 & -0.4 & -0.11 \end{array}\right]$$

$$\frac{R_2}{0.4} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 0.275 \\ 0 & -0.4 & -0.11 \end{array}\right]$$

$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 + 0.4R_2 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 0.725 \\ 0 & 1 & 0.275 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$S_1 = 0.725$

$S_2 = 0.275$

$$S = \begin{array}{c} \begin{array}{cc} H & N \end{array} \\ [0.725 \quad 0.275] \end{array}$$

In the long run, approximately 72.5% of boxcars will be on the home tracks.