Section 7.1

15 Given $P = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$ and $S_0 = \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \end{array}$,

find S_2.

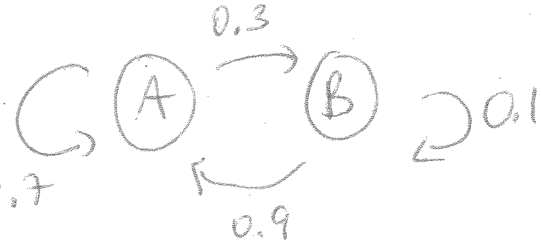
$$S_1 = S_0 P$$
$$= \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$
$$= \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$$

$$S_2 = S_1 P$$
$$= \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$
$$= \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.64 & 0.36 \end{bmatrix} \end{array}$$

The probability of being in State A after 2 time periods is 0.64

The probability of being in State B after 2 time periods is 0.36

(17) Given



and $S_0 = [0.2 \quad 0.8]$, find S_1.

$$P = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix} \end{array} \leftarrow \text{next state}$$

current state

$$S_1 = S_0 P$$

$$= [0.2 \quad 0.8] \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.86 & 0.14 \end{bmatrix} \end{array}$$

(23) Given same transition diagram as #17, and $S_0 = [0.9 \;\; 0.1]$, find S_2.

$$P = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix} \end{array}$$ (See Question #17)

$$S_1 = S_0 P$$
$$= [0.9 \;\; 0.1] \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix}$$
$$= [0.72 \;\; 0.28]$$

$$S_2 = S_1 P$$
$$= [0.72 \;\; 0.28] \begin{bmatrix} 0.7 & 0.3 \\ 0.9 & 0.1 \end{bmatrix}$$
$$= \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ [0.756 \;\; 0.244] \end{array}$$

(25) YES

It's square ✓

All entries ≥ 0 ✓

All rows sum to 1 ✓

(27) No

All entries ≥ 0 <u>is false</u>

All rows sum to 1 <u>is false</u>

(29) No

Matrix is <u>not</u> square.

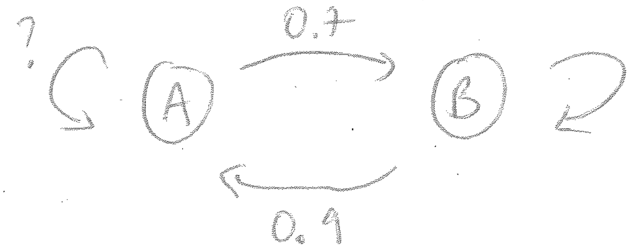
(31) YES

It's square ✓

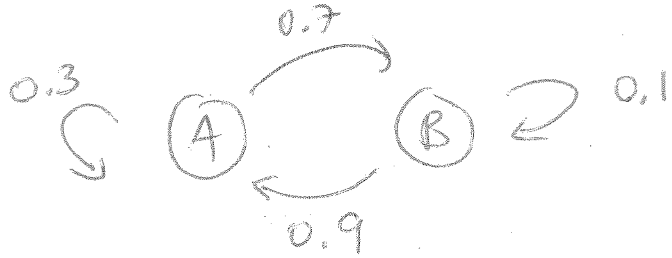
All entries ≥ 0 ✓

All rows sum to 1 ✓

(45)



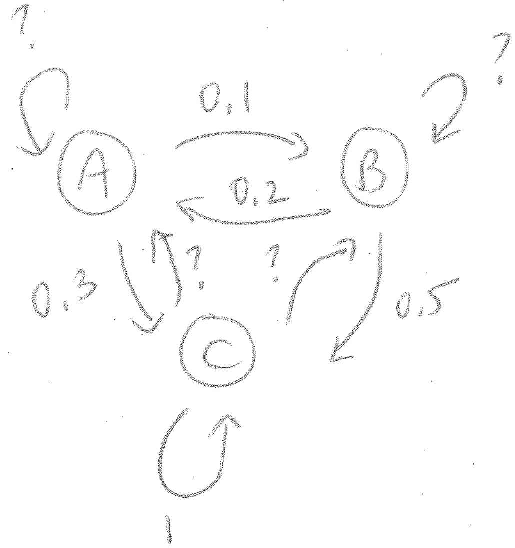
Arrows leaving a state must add up to 1 (so that the rows of the transition matrix sum to 1).



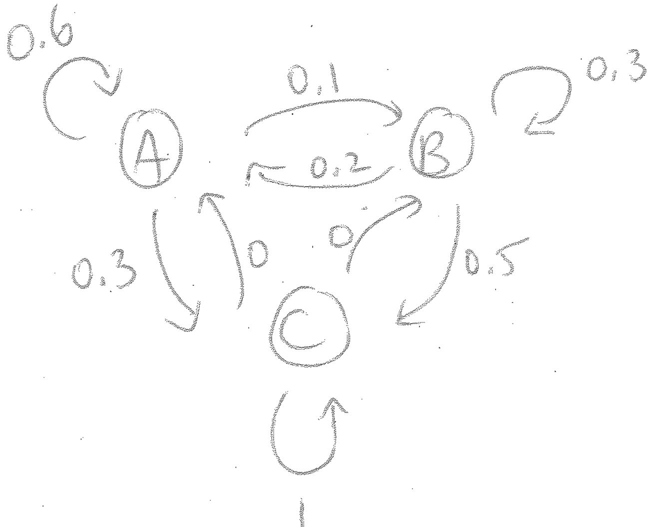
$$P = \begin{array}{c} \\ A \\ B \end{array} \!\! \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.3 & 0.7 \\ 0.9 & 0.1 \end{bmatrix} \end{array} \leftarrow \text{next state}$$

↑ current state

(47)



Arrows leaving a state must add up to 1 (so that the rows of the transition matrix sum to 1).



$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

(49) Given P, P^2 and P^3

$$P^2 = \begin{array}{c|ccc} & A & B & C \\ \hline A & & 0.35 & \\ B & & & \\ C & & & \end{array}$$

← future state

↑ current state

Probability of going from State A to State B in 2 trials is 0.35

use P^2

(51) Given P, P^2, P^3

$$P^3 = \begin{array}{c|ccc} & A & B & C \\ \hline A & & & \\ B & & & \\ C & 0.212 & & \end{array}$$

← future state

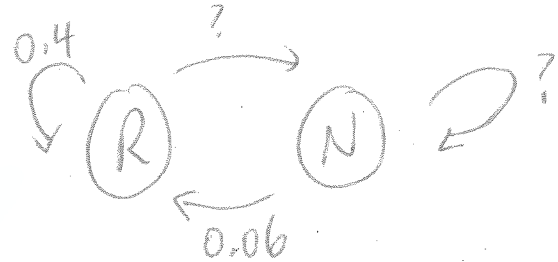
↑ current state

Probability of going from State C to State A in 3 trials is 0.212

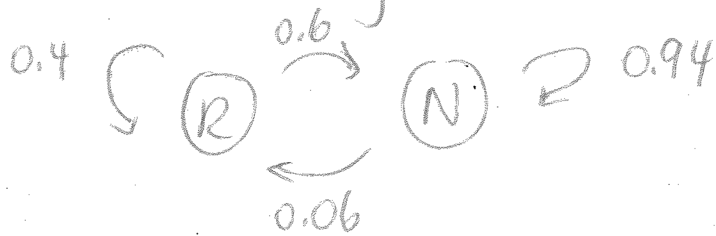
use P^3

(79) Let R = rain
N = no rain

a)



Arrows leaving a state sum to 1.



b)

$$P = \begin{array}{c c} & \begin{array}{cc} R & N \end{array} \\ \begin{array}{c} R \\ N \end{array} & \begin{bmatrix} 0.4 & 0.6 \\ 0.06 & 0.94 \end{bmatrix} \end{array}$$

$\leftarrow$ future state

$\uparrow$ current state

c)

Thursday $\quad S_0 = \begin{array}{c} \begin{array}{cc} R & N \end{array} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} \end{array}$

Friday $\quad S_1 = S_0 P$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.4 & 0.6 \\ 0.06 & 0.94 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} R & N \end{array} \\ \begin{bmatrix} 0.4 & 0.6 \end{bmatrix} \end{array}$$

$\rightarrow$

79 Cont'd

Saturday

$$S_2 = S_1 P$$

$$= \begin{bmatrix} 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 0.4 & 0.6 \\ 0.06 & 0.94 \end{bmatrix}$$

$$= \begin{array}{cc} R & N \\ \end{array}$$
$$\begin{bmatrix} 0.196 & 0.804 \end{bmatrix}$$

Probability that restaurant is closed on Saturday (due to rain) is 0.196

Sunday

$$S_3 = S_2 P$$

$$= \begin{bmatrix} 0.196 & 0.804 \end{bmatrix} \begin{bmatrix} 0.4 & 0.6 \\ 0.06 & 0.94 \end{bmatrix}$$

$$= \begin{array}{cc} R & N \\ \end{array}$$
$$\begin{bmatrix} 0.12664 & 0.87336 \end{bmatrix}$$

Probability that restaurant is closed on Sunday (due to rain) is 0.12664

81

Let $X' =$ not X

a)

$$\begin{tikzcd} X \arrow[loop left, "0.8"] \arrow[r, bend left, "0.2"] & X' \arrow[l, bend left, "0.2"] \arrow[loop right, "0.8"] \end{tikzcd}$$

b)

$$P = \begin{array}{cc} & \begin{array}{cc} X & X' \end{array} \\ \begin{array}{c} X \\ X' \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

$\leftarrow$ next state

current state

c) Now

$$S_0 = \begin{array}{c} \begin{array}{cc} X & X' \end{array} \\ \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \end{array}$$

1 week later

$$\begin{aligned} S_1 &= S_0 P \\ &= \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \\ &= \begin{array}{c} \begin{array}{cc} X & X' \end{array} \\ \begin{bmatrix} 0.32 & 0.68 \end{bmatrix} \end{array} \end{aligned}$$

32% will be using Brand X
1 week later.

$\rightarrow$

(81) Cont'd

2 weeks later

$$S_2 = S_1 P$$

$$= \begin{bmatrix} 0.32 & 0.68 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

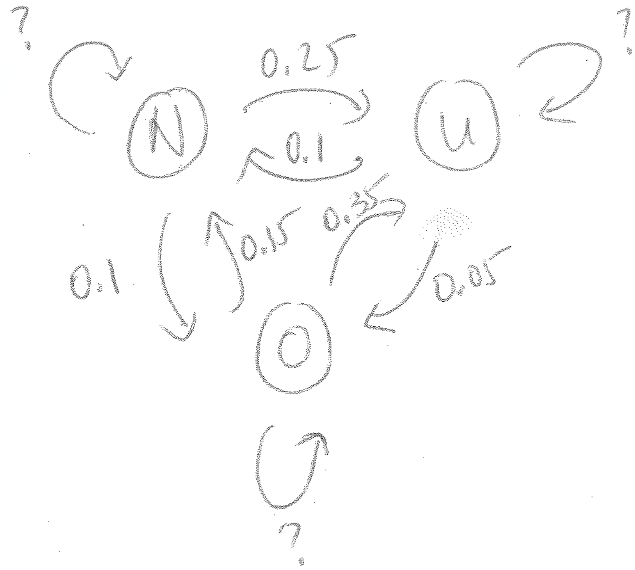
$$= \begin{array}{cc} X & X' \\ \left[0.392 \right. & \left. 0.608 \right] \end{array}$$

39.2% will be using Brand X
2 weeks later.

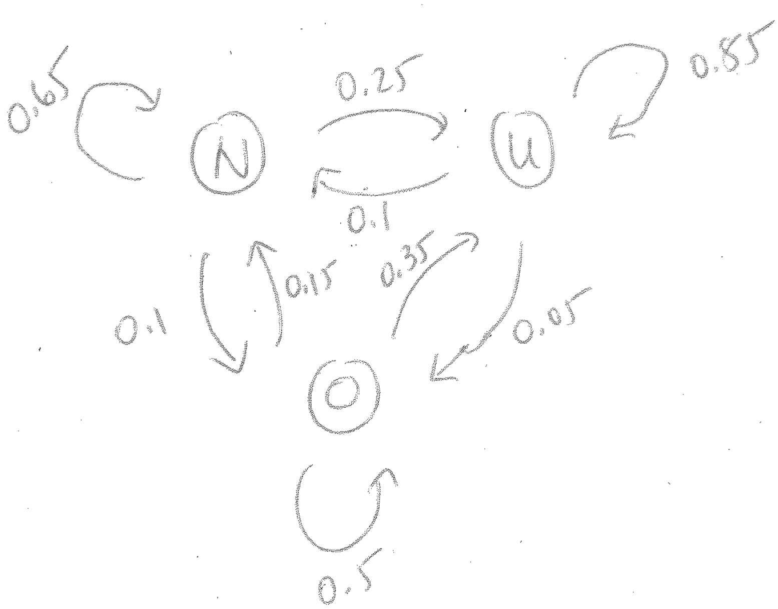
83

Let N = National Property
U = United Family
O = Other Companies

a)



Arrows leaving a state sum to 1.



b)

$$P = \begin{array}{c|ccc} & N & U & O \\ \hline N & 0.65 & 0.25 & 0.1 \\ U & 0.1 & 0.85 & 0.05 \\ O & 0.15 & 0.35 & 0.5 \end{array}$$

← next state

Current state

(83) Cont'd

c) and d) We are given S_0 in the question.

$$\text{Currently} \quad S_0 = \begin{array}{c} \begin{array}{ccc} N & U & O \end{array} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 \end{bmatrix} \end{array}$$

(the row sums to 1)

$$\text{Next year} \quad S_1 = S_0 P$$

$$= \begin{bmatrix} 0.5 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 0.65 & 0.25 & 0.1 \\ 0.1 & 0.85 & 0.05 \\ 0.15 & 0.35 & 0.5 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{ccc} N & U & O \end{array} \\ \begin{bmatrix} 0.385 & 0.45 & 0.165 \end{bmatrix} \end{array}$$

$$\text{The year after next} \quad S_2 = S_1 P$$

$$= \begin{bmatrix} 0.385 & 0.45 & 0.165 \end{bmatrix} \begin{bmatrix} 0.65 & 0.25 & 0.1 \\ 0.1 & 0.85 & 0.05 \\ 0.15 & 0.35 & 0.5 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{ccc} N & U & O \end{array} \\ \begin{bmatrix} 0.32 & 0.5365 & 0.1435 \end{bmatrix} \end{array}$$

Next year, National insures 38.5% and United insures 45%.

The year after next, National insures 32% and United insures 53.65%.