

Section 6.5

① $[A|I]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -6 & 1 & 0 & 0 \\ 3 & 4 & 7 & 0 & 1 & 0 \\ 4 & 4 & -22 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & -6 & 1 & 0 & 0 \\ 0 & 1 & 25 & -3 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \end{array} \right]$$

$$R_1 - R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & -31 & 4 & -1 & 0 \\ 0 & 1 & 25 & -3 & 1 & 0 \\ 0 & 0 & 2 & -4 & 0 & 1 \end{array} \right]$$

$$\frac{R_3}{2} \left[\begin{array}{ccc|ccc} 1 & 0 & -31 & 4 & -1 & 0 \\ 0 & 1 & 25 & -3 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0.5 \end{array} \right]$$

$$\begin{array}{l} R_1 + 31R_3 \\ R_2 - 25R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -58 & -1 & 15.5 \\ 0 & 1 & 0 & 47 & 1 & -12.5 \\ 0 & 0 & 1 & -2 & 0 & 0.5 \end{array} \right]$$

$\underbrace{\hspace{1.5cm}}_I \qquad \underbrace{\hspace{1.5cm}}_{A^{-1}}$

$$(2) \quad [A \mid I]$$

$$\left[\begin{array}{ccc|ccc} 4 & 2 & -1 & 1 & 0 & 0 \\ 7 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$R_1 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 7 & 2 & 1 & 0 & 1 & 0 \\ 4 & 2 & -1 & 1 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 - 7R_1 \\ R_3 - 4R_1 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 2 & -6 & 0 & 1 & -7 \\ 0 & 2 & -5 & 1 & 0 & -4 \end{array} \right]$$

$$\frac{R_2}{2} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & 0.5 & -3.5 \\ 0 & 2 & -5 & 1 & 0 & -4 \end{array} \right]$$

$$R_3 - 2R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & 0.5 & -3.5 \\ 0 & 0 & 1 & 1 & -1 & 3 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_3 \\ R_2 + 3R_3 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -2 \\ 0 & 1 & 0 & 3 & -2.5 & 5.5 \\ 0 & 0 & 1 & 1 & -1 & 3 \end{array} \right]$$

I

A^{-1}

(3)

 $[A | I]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & -1 & -3 & -2 & 1 & 0 \\ 0 & -2 & -7 & -3 & 0 & 1 \end{array} \right]$$

$$\frac{R_2}{-1} \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & -1 & 0 \\ 0 & -2 & -7 & -3 & 0 & 1 \end{array} \right]$$

$$R_1 - R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 3 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array} \right]$$

$$R_3 + 2R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 3 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right]$$

$$\frac{R_3}{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 3 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array} \right]$$

$$R_2 - 3R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 5 & -7 & 3 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array} \right]$$

$\underbrace{\quad\quad\quad}_I \quad \underbrace{\quad\quad\quad}_{A^{-1}}$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 5 & -7 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 12 \\ 10 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix} \quad \leftarrow -1(12) + 1(10) + 0(4)$$

$$\begin{array}{l} x = -2 \\ y = 2 \\ z = 4 \end{array}$$