Section 6.2

①

	x	y	z	#
	1	1	1	6
	2	3	2	14
	2	1	2	10

$R_2 - 2R_1$, $R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 0 & | & 2 \\ 0 & -1 & 0 & | & -2 \end{bmatrix}$$

$R_1 - R_2$, $R_3 + R_2$

$$\begin{bmatrix} \textcircled{1} & 0 & 1 & | & 4 \\ 0 & \textcircled{1} & 0 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$\boxed{z = \text{any value}}$

$x + z = 4 \Rightarrow \boxed{x = 4 - z}$

$\boxed{y = 2}$

(2)

$$\begin{array}{c} x \quad y \quad z \quad \# \\ \left[\begin{array}{ccc|c} 1 & -1 & 3 & 9 \\ -2 & 3 & -11 & -31 \\ 1 & -2 & 8 & 22 \end{array}\right] \end{array}$$

R_2+2R_1, R_3-R_1

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 9 \\ 0 & 1 & -5 & -13 \\ 0 & -1 & 5 & 13 \end{array}\right]$$

R_1+R_2

$$\left[\begin{array}{ccc|c} \textcircled{1} & 0 & -2 & -4 \\ 0 & \textcircled{1} & -5 & -13 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

R_3+R_2

$\boxed{z = \text{any value}}$

$x - 2z = -4 \Rightarrow \boxed{x = 2z - 4}$

$y - 5z = -13 \Rightarrow \boxed{y = 5z - 13}$

(3) three possible solution to Question (2):

Sub $z=0$: $\boxed{x=-4,\ y=-13,\ z=0}$

Sub $z=1$: $\boxed{x=-2,\ y=-8,\ z=1}$

Sub $z=2$: $\boxed{x=0,\ y=-3,\ z=2}$

④

$$\begin{array}{c} x \quad y \quad z \quad \# \\ \left[\begin{array}{ccc|c} 2 & 4 & 4 & 38 \\ 3 & 6 & 8 & 75 \\ 3 & 6 & 6 & 57 \end{array}\right] \end{array}$$

$\frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 19 \\ 3 & 6 & 8 & 75 \\ 3 & 6 & 6 & 57 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 19 \\ 0 & 0 & 2 & 18 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{R_2}{2}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 19 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 0 & 1 \\ 0 & 0 & \textcircled{1} & 9 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{y = \text{any value}}$

$x + 2y = 1 \Rightarrow \boxed{x = 1 - 2y}$

$\boxed{z = 9}$

(5)

$$\begin{array}{c} x \quad y \quad z \quad \# \\ \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ 7 & 8 & 9 & 19 \end{array}\right] \end{array}$$

$R_2 - 4R_1$, $R_3 - 7R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -3 \\ 0 & -6 & -12 & 12 \end{array}\right]$$

$\frac{R_2}{-3}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -6 & -12 & 12 \end{array}\right]$$

$R_1 - 2R_2$

$R_3 + 6R_2$

$$\left[\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 0 & 18 \end{array}\right]$$

The system has no solution.

(6)

$$\begin{array}{c} \\ \frac{R_1}{2} \\ \\ R_2-2R_1 \\ R_3-5R_1 \\ \\ \frac{R_2}{-5} \\ \\ R_1-4R_2 \\ \\ R_3+10R_2 \end{array} \quad \begin{array}{ccc|c} x & y & z & \# \\ \hline 2 & 8 & 10 & 18 \\ 2 & 3 & 4 & 2 \\ 5 & 10 & 13 & 13 \end{array} \quad \begin{array}{ccc|c} 1 & 4 & 5 & 9 \\ 2 & 3 & 4 & 2 \\ 5 & 10 & 13 & 13 \end{array} \quad \begin{array}{ccc|c} 1 & 4 & 5 & 9 \\ 0 & -5 & -6 & -16 \\ 0 & -10 & -12 & -32 \end{array} \quad \begin{array}{ccc|c} 1 & 4 & 5 & 9 \\ 0 & 1 & 1.2 & 3.2 \\ 0 & -10 & -12 & -32 \end{array} \quad \begin{array}{ccc|c} \textcircled{1} & 0 & 0.2 & -3.8 \\ 0 & \textcircled{1} & 1.2 & 3.2 \\ 0 & 0 & 0 & 0 \end{array}$$

z = any value

$y + 1.2z = 3.2 \Rightarrow$ $y = 3.2 - 1.2z$

$x + 0.2z = -3.8 \Rightarrow$ $x = -3.8 - 0.2z$

(7)

$$\begin{array}{c} \\ R_2-3R_1 \\ R_3-5R_1 \end{array} \quad \begin{array}{cc|c} x & y & \\ 1 & 2 & -4 \\ 3 & 4 & -14 \\ 5 & 6 & -24 \end{array} \quad \begin{bmatrix} 1 & 2 & -4 \\ 0 & -2 & -2 \\ 0 & -4 & -4 \end{bmatrix}$$

$$\frac{R_2}{-2} \quad \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & 1 \\ 0 & -4 & -4 \end{bmatrix}$$

$$\begin{array}{c} R_1-2R_2 \\ R_3+4R_2 \end{array} \quad \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow 0x+0y=0$$

NO INFO

$$\boxed{\begin{array}{l} x=-6 \\ y=1 \end{array}}$$

(8)

$$\begin{array}{c} x \quad y \\ \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 3 \\ 5 & 6 & 6 \end{array}\right] \end{array}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 5R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & 0 \\ 0 & -4 & 1 \end{array}\right]$$

$$\frac{R_2}{-2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 4R_2 \end{array} \left[\begin{array}{cc|c} & & \\ & & \\ 0 & 0 & 1 \end{array}\right]$$

The system has no solution.

(9)

$$\begin{array}{cc} & \begin{matrix} w & x & y & z \end{matrix} \\ \left[\begin{array}{cccc|c} 2 & 2 & 4 & 8 & 6 \\ 3 & 4 & 5 & 6 & 7 \end{array}\right] & \end{array}$$

$\frac{R_1}{2}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 3 \\ 3 & 4 & 5 & 6 & 7 \end{array}\right]$$

$R_2 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 3 \\ 0 & 1 & -1 & -6 & -2 \end{array}\right]$$

$R_1 - R_2$

$$\left[\begin{array}{cccc|c} \textcircled{1} & 0 & 3 & 10 & 5 \\ 0 & \textcircled{1} & -1 & -6 & -2 \end{array}\right]$$

$\boxed{y = \text{any value}}$, $\boxed{z = \text{any value}}$

$w + 3y + 10z = 5 \Rightarrow \boxed{w = 5 - 3y - 10z}$

$x - y - 6z = -2 \Rightarrow \boxed{x = -2 + y + 6z}$