1 a) $Pr(E) = 0.3 + 0.1$
$= 0.4$

b) $Pr(F) = 0.1 + 0.4$
$= 0.5$

c) $$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$
$$= \frac{0.1}{0.5}$$
$$= 0.2$$

d) $$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}$$
$$= \frac{Pr(E \cap F)}{Pr(E)}$$
$$= \frac{0.1}{0.4}$$
$$= 0.25$$

Note: $F \cap E = E \cap F$

③ a) $\Pr(E|F) = \dfrac{\Pr(E \cap F)}{\Pr(F)}$

$= \dfrac{0.1}{0.3}$

≈ 0.33

b) $\Pr(F|E) = \dfrac{\Pr(F \cap E)}{\Pr(E)}$

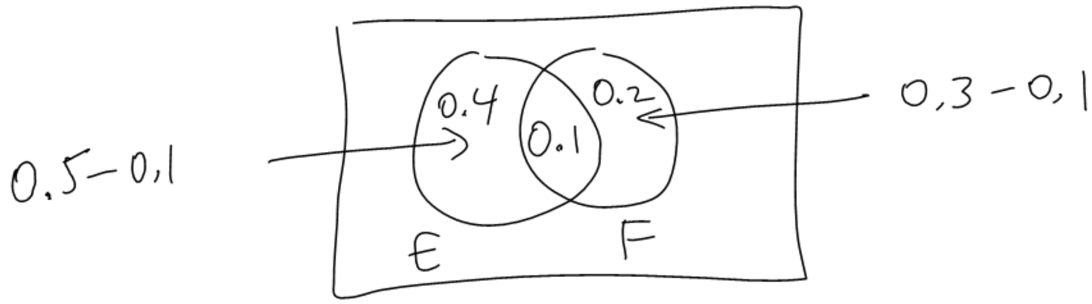
$= \dfrac{\Pr(E \cap F)}{\Pr(E)}$

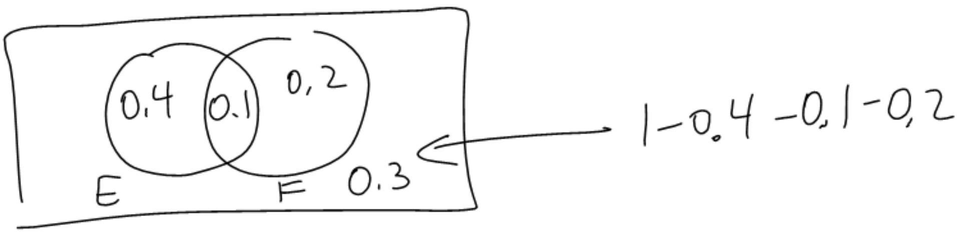
Note: $F \cap E = E \cap F$

$= \dfrac{0.1}{0.5}$

$= 0.2$

c) Let's draw a Venn diagram





→

③ c) continued

$$Pr(E|F') = \frac{Pr(E \cap F')}{Pr(F')}$$

$$= \frac{0.4}{(0.4+0.3)}$$

$$\approx 0.57$$

d) Refer to Venn diagram in part c).

$$Pr(E'|F') = \frac{Pr(E' \cap F')}{Pr(F')}$$

$$= \frac{0.3}{(0.4+0.3)}$$

$$\approx 0.43$$

(17) Let E: earn more than \$45,000/year
C: have college degree

$$Pr(E) = 0.25$$

$$Pr(E \cap C) = 0.1$$

Want $Pr(C \mid E)$

given

$$Pr(C \mid E) = \frac{Pr(C \cap E)}{Pr(E)}$$

$$= \frac{Pr(E \cap C)}{Pr(E)}$$

$\boxed{\text{Note: } C \cap E = E \cap C}$

$$= \frac{0.1}{0.25}$$

$$= 0.4$$

"Of the high-earners, 40% have a college degree."

21 a) $Pr(\text{officer}) = \dfrac{n(\text{officer})}{n(S)}$

$$= \frac{236.3}{1399.4}$$

$$\approx 0.17$$

b) $Pr(\text{Marine}) = \dfrac{n(\text{Marine})}{n(S)}$

$$= \frac{199.0}{1399.4}$$

$$\approx 0.14$$

c) $Pr(\text{is an officer in the Marines})$

$= Pr(\text{officer and Marine})$

$$= \frac{n(\text{officer} \cap \text{Marine})}{n(S)}$$

$$= \frac{22.1}{1399.4}$$

$$\approx 0.02$$

21 Cont'd

d) $Pr(\text{officer} \mid \text{Marine})$

given

$$= \frac{n(\text{officer} \cap \text{Marine})}{n(\text{Marine})}$$

$$= \frac{22.1}{199.0}$$

$$\approx 0.11$$

e) $Pr(\text{Marine} \mid \text{officer})$

given

$$= \frac{n(\text{Marine} \cap \text{officer})}{n(\text{officer})}$$

$$= \frac{22.1}{236.3}$$

$$\approx 0.09$$

(29) Let F: female

" H: majoring in health sciences

$Pr(F) = 0.54$

$Pr(H | F) = 0.09$

given

Want $Pr(F \cap H)$.

$$Pr(H|F) = \frac{Pr(H \cap F)}{Pr(F)}$$

$$0.09 = \frac{Pr(H \cap F)}{0.54}$$

$$0.09(0.54) = Pr(H \cap F)$$

$$0.05 \approx Pr(H \cap F)$$

$$Pr(H \cap F) \approx 0.05$$

$$\boxed{\text{Note: } H \cap F = F \cap H}$$

$$Pr(F \cap H) \approx 0.05$$

(33) Let's find $Pr(E \cap F)$.

Then we'll check if $Pr(E \cap F) = Pr(E) \cdot Pr(F)$

Inclusion-Exclusion Principle

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$

$$0.7 = 0.4 + 0.5 - Pr(E \cap F)$$

$$-0.2 = -Pr(E \cap F)$$

$$0.2 = Pr(E \cap F)$$

$$Pr(E \cap F) = 0.2$$

$$Pr(E \cap F) = Pr(E) \cdot Pr(F) \quad ?$$

$$0.2 = 0.4\,(0.5) \quad ?$$

YES

Yes, E and F are independent.

(35) $Pr(E \cap F) = Pr(E) \cdot Pr(F)$ because E and F are <u>independent</u>.

$$Pr(E \cap F) = 0.5(0.6)$$
$$= 0.3$$

Inclusion-Exclusion Principle

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$

$$Pr(E \cup F) = 0.5 + 0.6 - 0.3$$

$$Pr(E \cup F) = 0.8$$

(37) Because E and F are <u>independent</u>, $Pr(E|F) = Pr(E)$.

E and F represent any events so we can rename E and F.

$$Pr(A|B) = Pr(A)$$
$$Pr(M|N) = Pr(M) \text{ etc.}$$

This means $Pr(F|E) = Pr(F)$

$$Pr(F) = Pr(F|E)$$
$$Pr(F) = 0.6$$

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$$Pr(E) = 1 - Pr(E')$$
$$Pr(E) = 1 - 0.6$$
$$Pr(E) = 0.4$$

Since E and F are <u>independent</u>,

$$Pr(E \cap F) = Pr(E) \cdot Pr(F)$$
$$0.1 = (0.4)\ Pr(F)$$

$$\frac{0.1}{0.4} = Pr(F)$$

$$0.25 = Pr(F)$$
$$Pr(F) = 0.25$$

(43) We want to check if

$$Pr(E \cap F) = Pr(E) \cdot Pr(F)$$

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

$F = \{3, 6\}$

$E \cap F = \{6\}$

$$Pr(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{6}$$

$$Pr(E) = \frac{n(E)}{n(S)} = \frac{3}{6}$$

$$Pr(F) = \frac{n(F)}{n(S)} = \frac{2}{6}$$

$$Pr(E \cap F) = Pr(E) \cdot Pr(F) \quad ?$$

$$\frac{1}{6} = \frac{3}{6} \cdot \frac{2}{6} \quad ?$$

YES

Yes, E and F are independent.

(S3) 30% of those over 50 own a smartphone

70% " don't own a smartphone.

Let P_1 : Person 1 doesn't own a smartphone

" P_2 : " 2 "

" P_3 : " 3 "

" P_4 : Person 4 "

$Pr(P_1) = 0.7$, $Pr(P_2) = 0.7$, $Pr(P_3) = 0.7$

and $Pr(P_4) = 0.7$ from above.

We want $Pr(P_1 \cap P_2 \cap P_3 \cap P_4)$

$= Pr(P_1) \cdot Pr(P_2) \cdot Pr(P_3) \cdot Pr(P_4)$

The events are independent because the people were randomly chosen.

$= 0.7(0.7)(0.7)(0.7)$

≈ 0.24