(1) a) $\boxed{2} \times \boxed{2} \times \cdots \times \boxed{2}$

$\boxed{2}$: # of options for 1st toss (heads or tails)

$\boxed{2}$: # of options for 2nd toss

$\boxed{2}$: 8th toss

$= 2^8$

$= 256$

b) Choose which tosses are heads (the other tosses are forced to be tails). This is an <u>unordered</u> selection.

$C(8, 3) = 56$

Note: To convince yourself that the selection is <u>unordered</u>, say we choose tosses 1, 3, 7. That means H T H T T T H T.

Say we choose tosses 3, 7, 1.

That still means H T H T T T H T.

Order does <u>not</u> matter when selecting the tosses.

(3) a) $C(7,5)$ + $C(7,6)$ + $C(7,7)$

# of ways to get 5 heads OR # of ways to get 6 heads OR # of ways to get 7 heads

$= 21 + 7 + 1$

$= 29$

b) 2^7 — 29

Total # of ways to toss a coin 7 times

# of ways to get at least five heads (see part a)

$= 128 - 29$

$= 99$

Alternatively: $C(7,0) + C(7,1) + C(7,2) + C(7,3) + C(7,4)$

$= 99$

(7) $C(10,5)$ × $C(5,4)$ × 1

$C(10,5)$: # of ways to choose 5 of the 10 games to be wins (unordered selection)

$C(5,4)$: # of ways to choose 4 of the remaining 5 games to be losses

1: last game forced to be a tie

$= 252 \times 5 \times 1$

$= 1260$

Alternatively you could have chosen losses, then tie, then wins etc.

$C(10,4)$ × $C(6,1)$ × 1

$C(10,4)$: Choose 4 of 10 games to be losses

$C(6,1)$: Choose 1 of remaining 6 games to be a tie

1: Remaining 5 games are forced to be wins

$= 210 \times 6 \times 1$

$= 1260$

(9) Shortest route from A to B in Figure 2 is:

2 blocks south and 5 blocks east, in any order.

$$C(7,2) = 21$$

Choose 2 of 7 steps to be South (other steps forced to be east).

Alternatively: $C(7,5) = 21$

Choose 5 of 7 steps to be east (other steps forced to be south).

(11) Look at Figure 3.

$C(5,2)$ × $C(4,2)$

# of shortest routes from A to C (choose 2 of 5 steps to be South; other steps forced to be east)

# of shortest routes from C to B (choose 2 of 4 steps to be South; other steps forced to be east)

$= 10 \times 6$

$= 60$

(13) Look at Figure 2.

We can rephrase the question as:

How many sequences of 2 S's and 5 E's do not have consecutive S's?

E E E E E

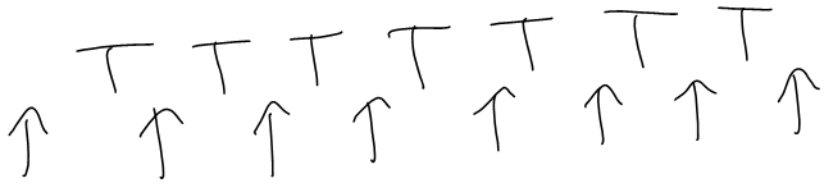
↑ ↑ ↑ ↑ ↑ ↑

Choose 2 of 6 spots to put the S's.

$C(6,2) = 15$

(17) Similar to Question 13.

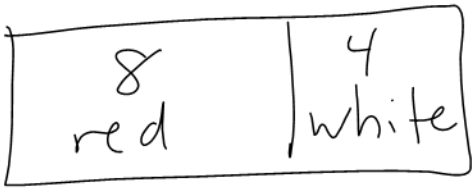
How many sequences of 3 H's and 7 T's do not have 6nsecutive H's?



Choose 3 of 8 spots to put the H's.

$C(8,3) = 56$

(21) Unordered selection



a) $C(12,4) = 495$

# of ways to choose 4 of 12 balls

b) $C(8,4) = 70$

# of ways to choose 4 of 8 red balls

c) $C(8,2) \times C(4,2)$

# of ways to Choose 2 of 8 red balls

# of ways to choose 2 of 4 white balls

$= 28 \times 6$

$= 168$

→

(21) Cont'd

d) $C(8,3) \times C(4,1) + C(8,4)$

Choose 3 red AND choose 1 white OR Choose 4 red

$= 56 \times 4 + 70$

$= 294$

(25) Unordered selection

$C(4,2) \times C(5,2)$

# of ways to choose 2 of 4 males

# of ways to choose 2 of 5 females

$= 6 \times 10$

$= 60$

(27)

$C(4,3)$ × $C(4,2)$

# of ways to choose 3 of 4 suits for the aces

# of ways to choose 2 of 4 suits for the kings

$= 4 \times 6$

$= 24$

(29)

13 × 12

# of options for denomination of the triple (A, 2, 3, ..., K)

# of options for denomination of the pair (must be different than the triple)

× $C(4,3)$ × $C(4,2)$ $= 3,744$

Choose suits for the triple

Choose suits for the pair

(35) Call the 3 women

W_1, W_2, W_3.

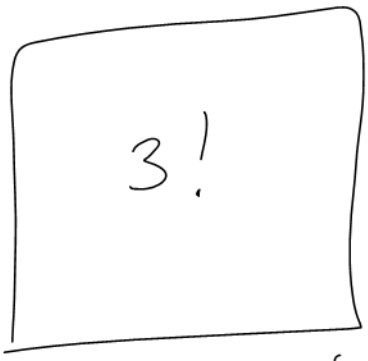
Call the 6 men

$M_1, M_2, \ldots, M_6$.

Glue the women together to create one object: $\boxed{W_1 W_2 W_3}$.

# of ways to order the 7 objects $M_1, M_2, \ldots, M_6, \boxed{W_1 W_2 W_3}$

$\times$



# of ways to order W_1, W_2, W_3

$= 5040 \times 6$

$= 30,240$