1. [2 marks] A company van is valued at $y = 52,500 - 1620x$, where y is in dollars and x is the number of years after 2015.

a) What was the value in the year 2021?

$x = 2021 - 2015$
$= 6$

$x = 6 \rightarrow y = 52500 - 1620x$
$y = 52500 - 1620(6)$
$y = 42780$

$\boxed{\$42,780}$

b) In which year will the value be \$8,760?

$y = 8760 \rightarrow y = 52500 - 1620x$
$8760 = 52500 - 1620x$
$-43740 = -1620x$

$\frac{-43740}{-1620} = x$

$x = 27$ Year $= 2015 + 27 = 2042$

2. [2 marks] Events E and F are independent.
Calculate $Pr(E)$ in each situation.
a) $Pr(E|F) = 0.42$ and $Pr(F) = 0.6$

$Pr(E) = Pr(E|F) = 0.42$

b) $Pr(E \cap F) = 0.42$ and $Pr(F) = 0.6$

$Pr(E \cap F) = Pr(E)\,Pr(F)$
$0.42 = Pr(E)(0.6)$

$\frac{0.42}{0.6} = Pr(E)$

$Pr(E) = 0.7$

3. [4 marks] A passcode consists of 5 digits, each chosen from 1 through 9. Find the probability that a passcode:

a) has no repeated digits

$n(S) = 9 \times 9 \times 9 \times 9 \times 9 = 9^5$

$n(E) = P(9,5)$

$Pr(E) = \frac{P(9,5)}{9^5} \approx 0.26$

b) does have repeated digits

$\approx 1 - 0.26$

≈ 0.74

c) is a palindrome (reads the same forwards as backwards)

$n(E) = 9 \times 9 \times 9 \times 1 \times 1 = 9^3$

(1 ← forced, 1 ← forced)

$Pr(E) = \frac{9^3}{9^5} \approx 0.01$

4. [2 marks] At a certain company: 35% of employees work in accounting, 21% of employees are runners, and 14% of employees work in accounting and are runners. Find the probability that an employee who works in accounting is a runner.

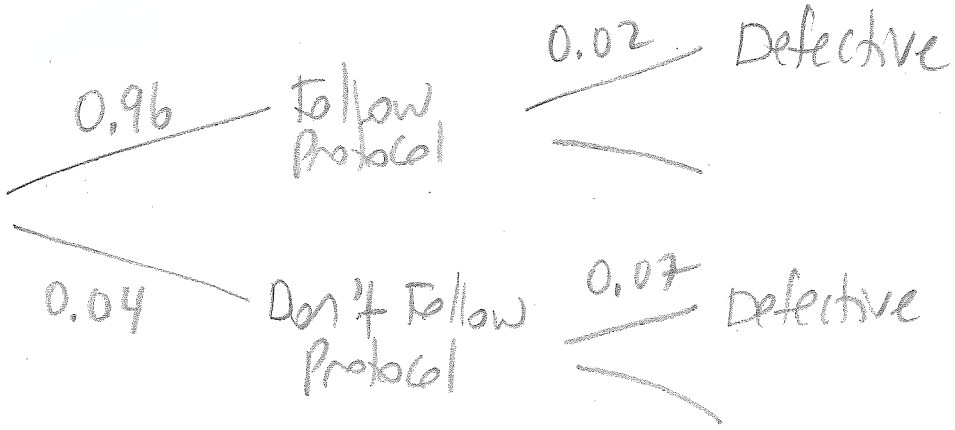
$Pr(\text{runner} \mid \text{accounting})$

$= \frac{Pr(\text{runner and accounting})}{Pr(\text{accounting})}$

$= \frac{0.14}{0.35}$

$= 0.4$

5. [4 marks] Employees at a manufacturing company follow protocol 96% of the time. When protocol is followed, 2% of items are defective.
When protocol is not followed, 7% of items are defective. What is the probability that protocol was followed, given that an item is defective?



$$\Pr(\text{Follow Protocol} \mid \text{Defective})$$

$$= \frac{\Pr(\text{Follow Protocol and Defective})}{\Pr(\text{Defective})}$$

$$= \frac{0.96(0.02)}{[0.96(0.02) + 0.04(0.07)]}$$

$$\approx 0.87$$

6. [3 marks] A box contains five \$10 bills and three \$20 bills. You pay \$6 and randomly draw a bill from the box. Let X be your net winnings (in dollars).

a) Find the probability distribution of X.

X = amount won - cost

	x	$P(x)$
get a \$10 bill →	4	5/8
get a \$20 bill →	14	3/8

b) Find the expected value of X.

$$E(x) = 4\left(\frac{5}{8}\right) + 14\left(\frac{3}{8}\right)$$

$$= 7.75$$

or $\mu = 7.75$

You expect net winnings of \$7.75, on average.

7. [3 marks] A multiple-choice test has six questions, with four possible answers for each question. A student guesses randomly on each question. Find the probability that the student gets at least two questions right.

Binomial Experiment

$n=6$

$p = P(\text{guess correctly on one question}) = \frac{1}{4}$

$q = 1-p = \frac{3}{4}$

$x =$ # of questions student gets correct

$$P(x \geq 2) = P(x=2) + P(x=3) + \dots + P(x=6)$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - 6C0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^6 - 6C1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^5$$

$$\approx 0.47$$

$nCx\, p^x q^{n-x}$