4.4 Conditional Probability and Independence

The probability of E, given that F occurs is written $Pr(E|F)$

"probability of E, given F"

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

or $\dfrac{n(E \cap F)}{n(F)}$ if all outcomes are equally likely

Ex:

Degree \ Job	Yes	No
Arts	87	13
Business	198	2

a) Find $Pr(\text{Job} | \text{Business})$

$$= \frac{n(\text{Job} \cap \text{Business})}{n(\text{Business})}$$

$$= \frac{198}{200}$$

$$= 0.99$$

Of those who majored in business, 99% have a job.

Notice: "Business" is the new sample space.

b) Find $Pr(\text{Business} \mid \text{Job})$

$$= \frac{n(\text{Business} \cap \text{Job})}{n(\text{Job})}$$

$$= \frac{198}{285}$$

$$\approx 0.69$$

<u>Ex</u>: A coin is tossed 3 times.

E: 3 heads appear

F: at least 2 heads appear Find $Pr(E|F)$

$E = \{HHH\}$

$F = \{HHT, HTH, THH, HHH\}$

$E \cap F = \{HHH\}$

$$Pr(E|F) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{1}{4}$$

<u>Ex</u>: 60% of employees are college grads
50% " permanent
20% permanent and college grads

Find the probability that an employee who is permanent is a college grad

Want Pr(College Grad | Permanent) ← given

$$= \frac{\text{Pr(College Grad and Permanent)}}{\text{Pr(Permanent)}}$$

$$= \frac{0.2}{0.5}$$

$$= 0.4$$

$$\boxed{Pr(E \cap F) = Pr(F) \cdot Pr(E|F)}$$

Ex: 40% of employees are permanent
30% of permanent employees have a pension

Pr(a random employee is permanent and has a pension)?

$Pr(\text{permanent}) = 0.4$

$Pr(\text{pension} \mid \text{permanent}) = 0.3$

$$\begin{aligned} Pr(\text{pension and perm}) &= Pr(\text{perm}) \cdot Pr(\text{pension} \mid \text{perm}) \\ &= 0.4\,(0.3) \\ &= 0.12 \end{aligned}$$

Events E and F are <u>independent</u> if the occurrence of one does not change the probability of the other.

<u>Ex</u>: Some <u>independent</u> events :

1) two tosses of a coin
2) two rolls of a die

<u>Ex</u>: Some <u>dependent</u> events :

1) weather today and weather tomorrow
2) price of gold today and price next week

Three equivalent statements:
1) E and F are independent
2) $Pr(E\mid F) = Pr(E)$
3) $Pr(E \cap F) = Pr(E) \cdot Pr(F)$

- If one statement is true then they're all true.
- " false then " false.

Ex: $Pr(E) = 0.6$

Find $Pr(E|F)$ if

a) E and F are independent

$$Pr(E|F) = Pr(E) = 0.6$$

b) E and F are not independent

Not enough info.

Ex: Are E and F independent?

$Pr(E) = 0.4$ $\quad Pr(F) = 0.3$ $\quad Pr(E \cup F) = 0.5$

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$
$$0.5 = 0.4 + 0.3 - Pr(E \cap F)$$
$$-0.2 = -Pr(E \cap F)$$
$$Pr(E \cap F) = 0.2$$

$$Pr(E \cap F) \neq Pr(E) \cdot Pr(F)$$ $\boxed{\text{No}}$

Ex: $Pr(E) = 0.6$ $Pr(F) = 0.3$

Find $Pr(E \cap F)$ if:

a) E and F are independent

$$Pr(E \cap F) = Pr(E)\,Pr(F) = 0.18$$

b) E and F are not independent

Not enough info.

Ex: A plane has 2 independent engines.

Engine 1 fails on 3% of flights

Engine 2 " 2% "

Pr(at least one engine works properly on next flight) ?

$= 1 - Pr(\text{both fail})$

$= 1 - Pr(\#1 \text{ fails}) \cdot Pr(\#2 \text{ fails})$ $\boxed{\text{INDEPENDENCE}}$

$= 1 - 0.03\,(0.02)$

$= 0.9994$

A set of 3 events is independent if:

$Pr(E_1 \cap E_2 \cap E_3) = Pr(E_1) \cdot Pr(E_2) \cdot Pr(E_3)$

Similar formulas for more than 3 events

Ex: A product has 3 independent parts.

In a year of use:

Part A fails 1% of the time

B 2%

C 4%

Pr(no part fails in a year of use)?

A: A fails $\qquad Pr(A) = 0.01$

A': A does not fail $\qquad Pr(A') = 0.99$

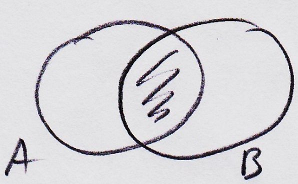
Want $Pr(A' \cap B' \cap C')$ $\qquad$ $\boxed{\text{INDEPENDENCE}}$

$= Pr(A') \cdot Pr(B') \cdot Pr(C')$

$= 0.99\,(0.98)\,(0.96)$

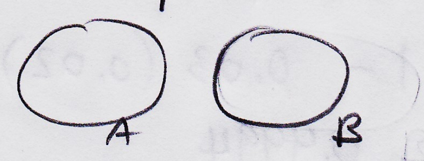
≈ 0.93

A and B are independent



$Pr(A \cap B) = Pr(A)\,Pr(B)$

A and B are mutually exclusive



$Pr(A \cap B) = 0$