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5.1 Expected Value

Notation: Probability of an event X

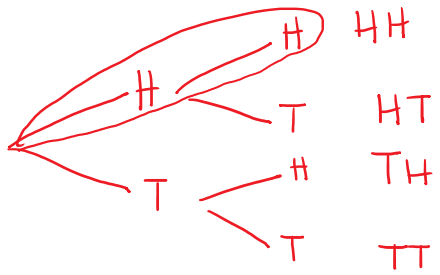
$Pr(X)$ or $P(X)$

Ex: A coin is flipped twice.

X = # heads that appear.

Find the probability distribution of X.

X	Description	# of Outcomes	P(X)
0	TT	1	1/4
1	HT, TH	2	2/4
2	HH	1	1/4
		Total = 4	



X	P(X)
0	0.25
1	0.5
2	0.25

Expected value of X: Theoretical average if the experiment were repeated infinitely-many times.

Ex: Coin example.

Find the expected value.

From above:

X	$P(X)$
0	0.25
1	0.5
2	0.25

$$E(X) = 0(0.25) + 1(0.5) + 2(0.25)$$
$$= 1$$

Expect 1 head in 2 coin tosses, on average.

<u>Ex</u>: A box contains 13 \$5 bills and 12 \$20 bills.

You pay \$11 and draw a bill.

Find your <u>expected</u> net winnings.

X = net winnings (\$)

= amount won − amount paid

	X = net winnings	$P(X)$
get \$5	$5-11=-6$	$\frac{13}{25}$
get \$20	$20-11=9$	$\frac{12}{25}$

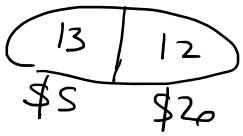
13	12
\$5	\$20

expected net winnings $E(X) = -6\left(\frac{13}{25}\right) + 9\left(\frac{12}{25}\right)$

$= 1.2$

Expect to win \$1.20, on average.

Ex: Same box of bills



You pay \$k and draw a bill.
Find k so that you expect to win \$3.

X = net winnings (\$)

	X	P(X)
get \$5	$5-k$	$\frac{13}{25}$
get \$20	$20-k$	$\frac{12}{25}$

Given $E(X) = 3$

$$(5-k)\frac{13}{25} + (20-k)\frac{12}{25} = 3$$

Multiply by 25:

$$(5-k)\frac{13}{\cancel{25}}\cancel{25} + (20-k)12 = 75$$

$$65 - 13k + 240 - 12k = 75$$

$$-25k = \cancel{75 - 65 - 240}\ -230$$

$$k = \frac{-230}{-25} = 9.2$$

$$\boxed{k = \$9.20}$$

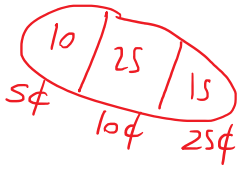
Ex: A bowl contains 10 nickels

and 25 dimes
15 quarters.

You draw a coin.
What is the expected value of the coin?

X = value of the coin ($)

X	$P(X)$
0.05	10/50
0.10	25/50
0.25	15/50



$$E(X) = 0.05\left(\frac{10}{50}\right) + 0.10\left(\frac{25}{50}\right) + 0.25\left(\frac{15}{50}\right)$$
$$= 0.135$$

Value of the coin is \$0.135, on average.