Week 9 Friday

March 8, 2019 8:30 AM

Quiz Tues 6.4

Omit Section 6.7 (Financial Math)

6.6 Logarithmic and Exponential Equations Cont'd

Recap 4 tools (see Wednesday notes)

Ex: Solve $2^{x+1} = 3^{x}$

Take ln of both sides:

$$\ln 2^{x+1} = \ln 3^{x}$$

Recall $\log_a M^r = r \log_a M$

$$(x+1)\ln 2 = x \ln 3$$

$$x\ln 2 + \ln 2 = x \ln 3$$

$$x\ln 2 - x\ln 3 = -\ln 2$$

$$x(\ln 2 - \ln 3) = -\ln 2$$

$$x = \frac{-\ln 2}{\ln 2 - \ln 3} \text{ or } \frac{\ln 2}{\ln 3 - \ln 2}$$

Ex: Solve $9^{x} - 2 \cdot 3^{x} - 24 = 0$ (TRICKY)

Notice $9^x = (3^2)^x = 3^{2x} = (3^x)^2$

Equation: $(3^x)^2 - 2\cdot 3^x - 24 = 0$

Sub $u = 3^x$

$$u^2 - 2u - 24 = 0$$

$$(u-6)(u+4) = 0$$

$$(3^x - 6)(3^x + 4) = 0$$

$3^x - 6 = 0$

$3^x = 6$

$\ln 3^x = \ln 6$

$x \ln 3 = \ln 6$

$$x = \frac{\ln 6}{\ln 3}$$

$3^x + 4 = 0$

$3^x = -4$

$\ln 3^x = \ln(-4)$

no solution

Answer: $x = \frac{\ln 6}{\ln 3}$

6.8 Exponential Growth and Decay

Exponential Growth

e.g. population

TYPICALLY

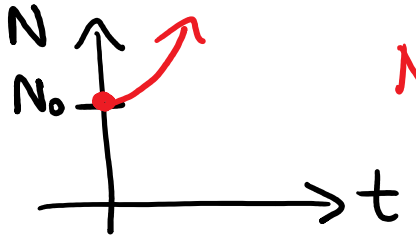
$$N = N_0 e^{kt}$$

N = population

N_0 = initial population

t = time

k = growth rate ($k > 0$)



$N = N_0 e^{kt}$

Formula Sheet: $A = Pe^{rt}$

A = Population

P = Initial Pop.

r = growth rate

t = time

Ex: Bacteria growth is given by $N = 6e^{0.038t}$ where N: grams and t: days.

Find:

a) initial mass

6 grams

b) growth rate

0.038

3.8% per day

c) mass after 10 days

$t=10 \rightarrow N = 6e^{0.038t}$

$= 6e^{[0.038 \times 10]}$

≈ 8.8 g

d) time when mass is 10

$N = 6e^{0.038t}$

$N=10$: $10 = 6e^{0.038t}$

Isolate the exponential

$\frac{10}{6} = e^{0.038t}$

$\ln\left(\frac{10}{6}\right) = \ln e^{0.038t}$

slicker $\ln e^{?} = ?$

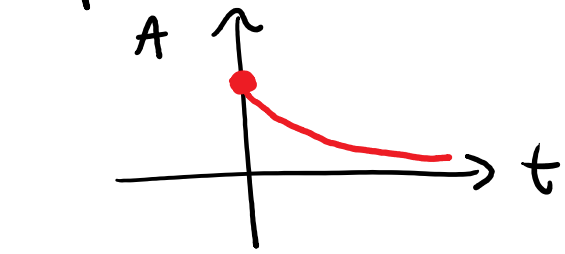
$\ln\left(\frac{10}{6}\right) = 0.038t \ln e$

$\ln e = 1$

$\frac{\ln\left(\frac{10}{6}\right)}{0.038} = t$

$t \approx 13$ days

Exponential Decay



$A = Pe^{rt}$ (Now $r < 0$)

As $t \to \infty$ $A = Pe^{-(\text{big})} \to 0$