Week 9 Wednesday

March 6, 2019 7:41 AM

Test tomorrow

6.5 Properties of Logarithms Cont'd

Omit 6.5 #91

Recap: Log Rules

1) $\log_a(MN) = \log_a M + \log_a N$
2) $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
3) $\log_a M^r = r \log_a M$

Ex: Write as a single logarithm

a) $\log_a 5 + 2\log_a 3$

$= \log_a 5 + \log_a 3^2$

$= \log_a (5 \cdot 3^2)$

$= \log_a 45$

b) $\frac{2}{3}\ln 27 - \ln(5-3)$

$= \ln 27^{2/3} - \ln 2$

$= \ln 9 - \ln 2$

$= \ln\left(\frac{9}{2}\right)$

Calculator can only compute with base 10 and base e

Change of Base Formula

$$\log_a M = \frac{\ln M}{\ln a}$$

(A)

Ex: Round to 2 decimal places

$\log_{\sqrt{2}} \frac{6}{5}$

$= \frac{\ln\left(\frac{6}{5}\right)}{\ln \sqrt{2}}$

≈ 0.53

Plan:

6.6 Solving Equations

6.8 Applications

(omit 6.7) Financial Math

6.6 Logarithmic and Exponential Equations

Ex: Solve $\log_3 (2-4x) = 2$

→ Exponential Form

$3^2 = 2-4x$

$9 = 2-4x$

$7 = -4x$

$-\frac{7}{4} = x$

> If the original equation involves logarithms, check that the solution is in domain
>
> $\log_b x$ has domain $x > 0$

Rephrased: Don't put negative # into log
" 0 "

For example above, need $2-4x > 0$

$2-4\left(-\frac{7}{4}\right) > 0$ ✓

Ex: Solve $\log_7 (x+2) + \log_7 (x+8) = 1$

Log Rule $\log_7 [(x+2)(x+8)] = 1$

→ Exponential Form

$7^1 = (x+2)(x+8)$

$7 = x^2 + 10x + 16$

$$0 = x^2 + 10x + 9$$

$$0 = (x+1)(x+9)$$

$$x = -1, -9$$

CAUTION

Domain of $\log_b x$ is $x > 0$

Need $x+2 > 0$ and $x+8 > 0$

$$\boxed{x = -1}$$

$x = -9$ is discarded

Ex: Solve $2\log_2 x = -\log_2 16$

$$\log_2 x = -\frac{1}{2}\log_2 16$$

Power Rule

$$\log_2 x = \log_2 16^{-1/2}$$

Bases are equal $\Rightarrow$ $x = 16^{-1/2}$

$$x = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

Check domain: Need $x > 0$

$$\boxed{x = \frac{1}{4}}$$

3 Tools for Solving

1) log frm $\leftrightarrow$ exponential frm

2) 3 log rules

3) equal bases

Ex: Solve $1.1 = 3.6\, e^{t+2}$

Isolate the exponential

$$\frac{1.1}{3.6} = e^{t+2}$$

Take ln of both sides:

$$\ln \frac{1.1}{3.6} = \underbrace{\ln e^{t+2}}_{\text{Power Rule}}$$

$$\ln \frac{1.1}{3.6} = (t+2)\underbrace{\ln e}_{1}$$

In general $\ln e^{y} = y$

$$\ln \frac{1.1}{3.6} = t+2$$

$$t = \ln\left(\frac{1.1}{3.6}\right) - 2$$

Summary: The 4 Tools for Solving Exp/Log Equations

1) log frm $\leftrightarrow$ exponential frm

2) 3 log rules

3) equal bases

4) take ln of both sides

Ex: Solve $2^{x+1} = 3^{x}$

Take ln: $\ln 2^{x+1} = \ln 3^{x}$

Power Rule: $(x+1)\ln 2 = x \ln 3$

$\vdots$