Week 8 Wednesday

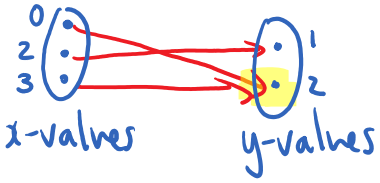
February 27, 2019 7:43 AM

6.2 One-to-One Functions and Inverse Functions

A function is one-to-one if different x-values have different y-values.

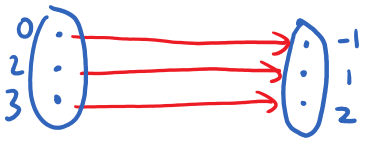
Ex: $f = \{(0,2), (2,1), (3,2)\}$

not one-to-one



Ex: $g = \{(0,-1), (2,1), (3,2)\}$

is one-to-one

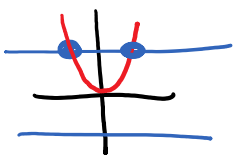


Horizontal Line Test:

If every horizontal line intersects a graph in at most 1 point, then the function is one-to-one.

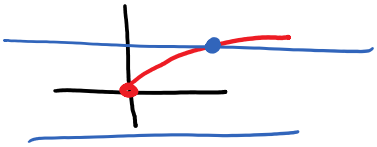
Ex: Is the function one-to-one?

a) $y = x^2$



not one-to-one

b) $y = \sqrt{x}$



one-to-one

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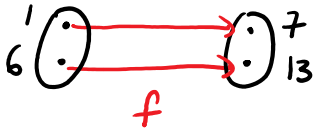
f^{-1} : the inverse of f

- Intuitively : f^{-1} undoes f
- f^{-1} is only defined when f is one-to-one

Ex: $f = \{(1,7), (6,13)\}$



Formally :

If f is the set of points (x, y)

then f^{-1} " (y, x)

Ex: $f = \{(1,7), (6,13)\}$

Domain of f $= \{1, 6\}$ = Range of f^{-1}

Range of f $= \{7, 13\}$ = Domain of f^{-1}

True in general

DEFINITION

Two functions f and g are <u>inverses</u> if:

$$f(g(x)) = x$$

and $$g(f(x)) = x$$

<u>Ex</u>: Verify that the inverse of $f(x) = \frac{1}{x+2}$ is $f^{-1}(x) = \frac{1}{x} - 2$

1) Check $f(f^{-1}(x)) = x$

$$f(f^{-1}(x)) = f\left(\frac{1}{x} - 2\right)$$

$$= \frac{1}{\left(\frac{1}{x} - 2\right) + 2}$$

$$= \frac{1}{\left(\frac{1}{x}\right)}$$

$$= \frac{x}{1} = x \quad \checkmark$$

2) Check $f^{-1}(f(x)) = x$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x+2}\right)$$

$f^{-1}(x) = \frac{1}{x} - 2$

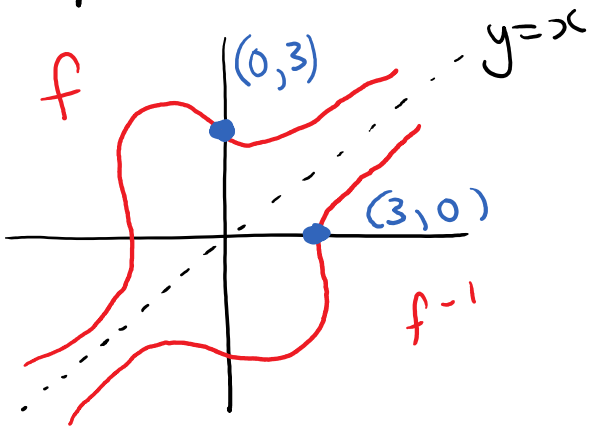
$$= \frac{1}{\boxed{\frac{1}{x+2}}} - 2$$

$$= \frac{x+2}{1} - 2$$

$$= x+2-2$$

$$= x \quad \checkmark$$

The graphs of f and f^{-1} are symmetric about the line $y = x$



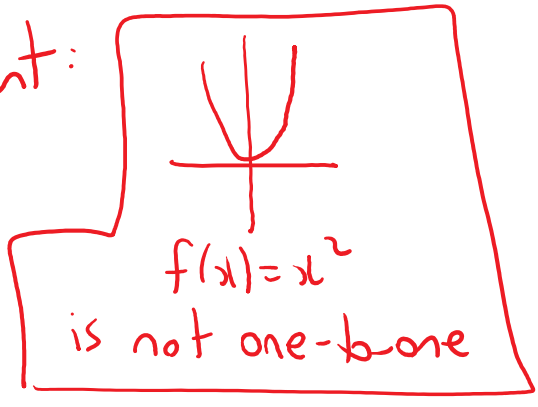
Why? x and y-values are swapped

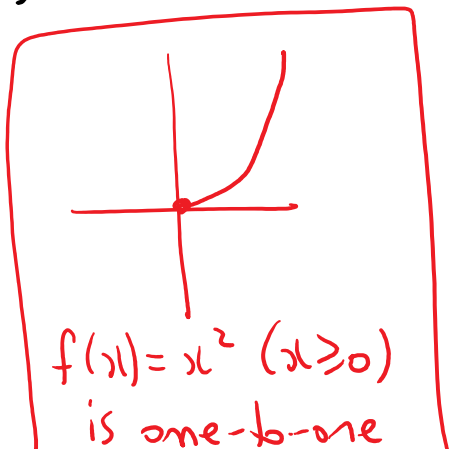
To find the formula for $f^{-1}(x)$, swap x and y in the formula for $f(x)$.

Ex: $f(x) = x^2 \quad (x \geq 0)$

Find a formula for $f^{-1}(x)$

Comment:





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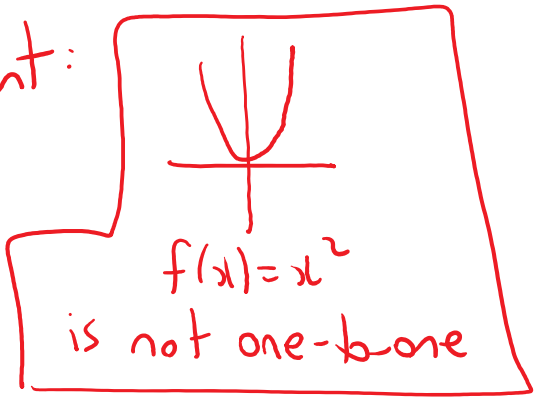
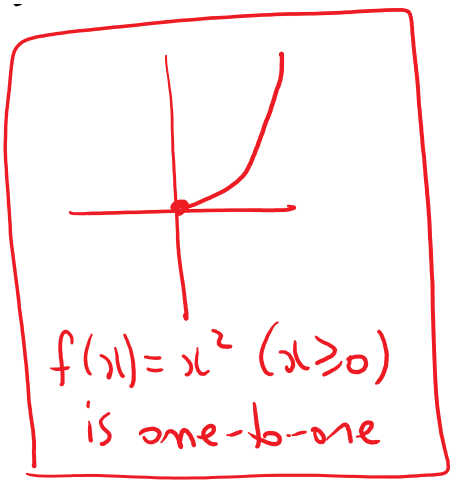


$f(x) = x^2$ is not one-to-one



$f(x) = x^2$ $(x \geq 0)$ is one-to-one

f: $y = x^2$ $(x \geq 0)$

Swap x and y

f^{-1}: $x = y^2$ $(y \geq 0)$

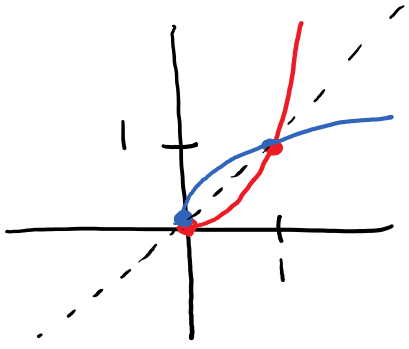
$y^2 = x$

$y = \pm\sqrt{x}$

Choose $+$ because $y \geq 0$

$y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$



$f = x^2$ $(x \geq 0)$

$f^{-1} = \sqrt{x}$