

Quiz tomorrow 5.2

Test Thurs March 7th

3.5, 3.6, 4.1, 4.3-4.5, 5.1-5.5, 6.1-6.2

8 Questions

Do Sugg HW

Do Practice Questions www.leahhoward.com

Exam Wed April 24th 1:30pm TEC 175

5.5 Real Zeros of a Polynomial Cnt'd

- Recap Remainder Theorem

- Factor Theorem

Let $f(x)$ be a polynomial

$x-c$ is a factor of $f(x)$ if and only if $f(c)=0$

Quick Ex:

If $x+4$ is a factor of $f(x)$ then $f(-4)=0$

If $f(6)=0$ then $f(x)=(x-6)(?)$

Ex: a) Remainder when $f(x)=x^4+17x^3+380$
is divided by $x+3$?

Dividing by $x-c$: Sub $x=c$

Dividing by $x-c$: Sub $x=c$
 $x+3$ $x=-3$

$$f(-3) = (-3)^4 + 17(-3)^3 + 380 \\ = 2$$

b) Is $x+3$ a factor of $f(x)$?

No $f(-3) \neq 0$
remainder $\neq 0$

A zero of a function f is a # so that $f(\#) = 0$

Rational Zeros Theorem

Given a polynomial with integer coefficients,
each rational zero is of the form:

$\frac{p}{q}$ $\leftarrow$ divides constant
 $\frac{p}{q}$ $\leftarrow$ divides leading coefficient

Ex: List the possible rational zeros of
 $f(x) = 2x^3 - 11x - 7$

Constant = -7
p divides -7
p: $\pm 1, \pm 7$

leading coeff. = 2
q divides 2

$$\boxed{\begin{array}{l} q \text{ divides } 2 \\ q: \pm 1, \pm 2 \end{array}}$$

$$\frac{p}{q}: \frac{\pm 1}{\pm 1}, \frac{\pm 7}{\pm 1}, \frac{\pm 1}{\pm 2}, \frac{\pm 7}{\pm 2}$$

$$\frac{p}{q}: \pm 1, \pm 7, \pm \frac{1}{2}, \pm \frac{7}{2}$$

Could check:

$$f(1) \neq 0$$

$$f(-1) \neq 0$$

$\vdots$

at least 1 # on the list will work

Ex: Factor $f(x) = x^3 + 5x^2 + 7x + 3$ fully

1) Find 1 zero using Rational Zeros Theorem

Constant = 3

$$p: \pm 1, \pm 3$$

leading coeff. = 1

$$q: \pm 1$$

$$\frac{p}{q}: \frac{\pm 1}{\pm 1}, \frac{\pm 3}{\pm 1}$$

$$\boxed{\pm 1, \pm 3}$$

$$f(1) \neq 0$$

$$f(-1) = (-1)^3 + 5(-1)^2 + 7(-1) + 3$$

$$= 0 \checkmark$$

-1 is a zero

2) Use Factor Theorem

$$f(x) = (x+1)(\quad?)$$

$$x^3 + 5x^2 + 7x + 3 = (x+1)g(x)$$

To find $g(x)$: $\frac{x^3 + 5x^2 + 7x + 3}{x+1} = g(x)$

Long Division

$$\begin{array}{r}
 \overline{x^2 + 4x + 3} \\
 (x+1) \overline{) x^3 + 5x^2 + 7x + 3} \\
 \underline{-(x^3 + x^2)} \\
 4x^2 + 7x + 3 \\
 \underline{-(4x^2 + 4x)} \\
 3x + 3 \\
 \underline{-(3x + 3)} \\
 0
 \end{array}$$

$$f(x) = (x+1)(x^2 + 4x + 3)$$

3) Factor fully

$$f(x) = (x+1)(x+1)(x+3)$$

$$\text{or } f(x) = (x+1)^2(x+3)$$

Ex: Follow-up

Ex: Follow-up

Solve $x^3 + 5x^2 + 7x + 3 = 0$

Factor: $(x+1)^2(x+3) = 0$

$$x = -1, -3$$

Reminder: $x^2 + a$ doesn't factor when $a > 0$

e.g. $(x-1)(x^2+4)$ is fully factored