5.4 Polynomial and Rational Inequalities Cont'd

Ex: Solve $x^3 - x^2 - 28x > 2x$

1) $y > 0$ or $y < 0$

$\underbrace{x^3 - x^2 - 30x}_{y} > 0$

2) Set $y = 0$

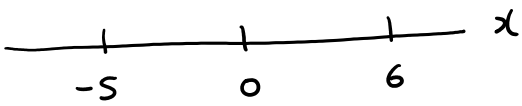
$x^3 - x^2 - 30x = 0$

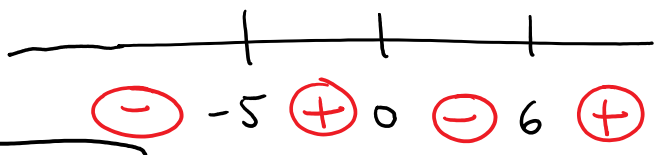
$x(x^2 - x - 30) = 0$

$x(x+5)(x-6) = 0$

$x = 0, -5, 6$

3) Number line





$y = x^3 - x^2 - 30x$

4) Answer

Want $y > 0$

$-5 < x < 0$ or $x > 6$

$(-5, 0) \cup (6, \infty)$

A rational function $f(x)$ can only change sign when $f(x)=0$ or $f(x)$ is undefined

Ex: Solve $\frac{4x-2}{x+1} \leq 1$

1) Get $y \leq 0$ or $y \geq 0$

$$\frac{4x-2}{x+1} - 1 \leq 0$$

Common denominator

$$\frac{4x-2}{x+1} - \frac{(x+1)}{x+1} \leq 0$$

$$y \rightarrow \boxed{\frac{3x-3}{x+1}} \leq 0$$

2) Where can y change sign?

$y=0$ or $y=$ undefined

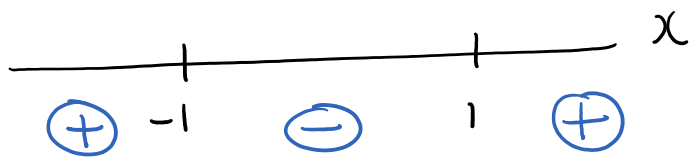
$$\frac{3x-3}{x+1} = 0$$

$$3x-3=0 \qquad x+1=0$$

$$3x=3 \qquad x=-1$$

$$x=1$$

3) Number line



$$y = \frac{3x-3}{x+1}$$

4) Answer

Want $y \leq 0$

Might be tempted to write $-1 \leq x \leq 1$ (incorrect)

CAUTION

Domain of y: $x \neq -1$

Final answer: $-1 < x \leq 1$

5.5 Zeros of a Polynomial

For example, where is $x^3 + 5x^2 + 7x + 3$ equal to 0?

NO/FEW TOOLS SO FAR

Ex: Find remainder when $f(x) = x^3 + 7x^2 - 36$ is divided by $x-5$

$$
\begin{array}{r}
x^2 + 12x + 60 \\
(x-5) \overline{)\, x^3 + 7x^2 + 0x - 36} \\
-(x^3 - 5x^2) \\
\hline
12x^2 + 0x - 36 \\
-(12x^2 - 60x) \\
\hline
60x - 36 \\
-(60x - 300) \\
\hline
264 \leftarrow \text{remainder}
\end{array}
$$

Remainder Theorem

Let $f(x)$ be a polynomial.
When $f(x)$ is divided by $x-c$ the remainder is $f(c)$.

Check for the long division example:

$f(x) = x^3 + 7x^2 - 36$

$c = 5$

Remainder is $f(c) = f(5) = 5^3 + 7(5^2) - 36$

$= 264$ ✓

This will help us find factors of $f(x)$

Big Goal: Solve $f(x) = 0$