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Quiz Tues 4.3

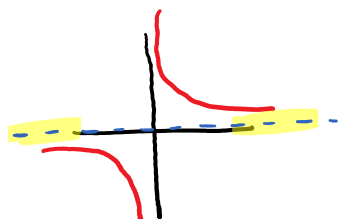
5.2 Properties of Rational Functions Gnt'd

- Vertical Asymptotes
- Horizontal "

The line $y=c$ is a horizontal asymptote
if $y \rightarrow c$ as $x \rightarrow \pm\infty$

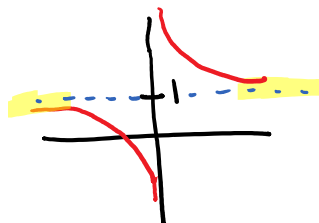
QUICK EX:

a) $y = \frac{1}{x}$



H.A. $y=0$

b) $y = \frac{1}{x} + 1$



H.A. $y=1$

Consider $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

3 cases for H.A. :

1) If degree (numerator) < degree (denominator) H.A. $y=0$

2) If $\quad \quad \quad =$ H.A. $y=c$
($c \neq 0$)

2) If

=

H.A. $y=c$
($c \neq 0$)

3) If

>

no H.A.

Ex: Find any H.A.

a) $y = \frac{4x^6 + 1}{x^9 - 2}$

as $x \rightarrow \pm\infty$ $y \rightarrow 0$

H.A. $y=0$

~~IN GENERAL~~

If degree (num.) < degree (denom.) then H.A. $y=0$

b) $y = \frac{-6x^5 + 3}{8x^5 - 7x^4}$

as $x \rightarrow \pm\infty$ $y \approx \frac{-6x^5}{8x^5} = -\frac{6}{8} = -\frac{3}{4}$

H.A. $y = -\frac{3}{4}$

IN MORE DETAIL

x	y
10	
100	
1000	
$\vdots$	$\vdots$
∞	$-\frac{3}{4}$

~~IN GENERAL~~

If degree (num.) = degree (denom.)
then take ratio of leading terms
H.A. $y=c$ ($c \neq 0$)

$$c) \quad y = \frac{2x^3 - 5}{x^2 + 2}$$

as $x \rightarrow \pm\infty$, $y \rightarrow \pm\infty$

No H.A.

IN GENERAL

If $\deg(\text{num.}) > \deg(\text{denom.})$
then No H.A.

Ex: Find any H.A.

$$a) \quad y = \frac{-7x^7 + 2}{14x^7 - 3x^2}$$

as $x \rightarrow \pm\infty$ $y \approx \frac{-7x^7}{14x^7} = -\frac{7}{14} = -\frac{1}{2}$

H.A. $y = -\frac{1}{2}$

$$b) \quad y = \frac{2x^3 - 100x^2}{1000x^4}$$

as $x \rightarrow \pm\infty$ $y \approx \frac{2x^3}{1000x^4} = \frac{1}{500x} \rightarrow 0$

H.A. $y = 0$

$$c) \quad y = \frac{x^5 + 2x^4}{x^4}$$

$$\text{as } x \rightarrow \pm \infty \quad y \approx \frac{x^5}{x^4} \approx x \rightarrow \pm \infty$$

No H.A.

5.3 Graphing Rational Functions

$$\underline{\text{Ex:}} \quad f(x) = \frac{2x^2 - 2x}{x^2 - x - 2}$$

1) Find x- and y-intercepts

y-intercept: set $x=0$

$$y = \frac{0}{-2} = 0$$

$$(x, y) = (0, 0)$$

x-intercepts: set $y=0$

$$0 = \frac{2x^2 - 2x}{x^2 - x - 2}$$

Mult. by $x^2 - x - 2$:

$$0 = 2x^2 - 2x$$

$$0 = 2x(x - 1)$$

$$0 = 2x(x-1)$$

$$x=0, 1$$

$$(x,y) = (0,0) \text{ and } (1,0)$$

2) Reduce $f(x)$ to lowest terms
V.A.?

$$f(x) = \frac{2x^2 - 2x}{x^2 - x - 2} \quad \text{FACTOR}$$

$$= \frac{2x(x-1)}{(x-2)(x+1)} \quad \checkmark$$

$$\text{V.A. } x=2, x=-1$$

Reduce $f(x)$ to lowest terms
V.A. are the zeros of the denom.

