5.1 Polynomial Functions Cont'd

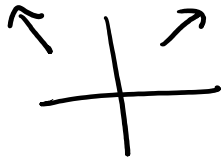
Recap from yesterday:

- End behaviour
- Multiplicity of zeros
- When a graph crosses/touches x-axis

Ex: Graph $f(x) = x^4 + 2x^3 - 24x^2$

1) End behaviour

$f(x) \approx x^4$ (as $x \to \pm\infty$)



2) Real zeros (x-intercepts) and multiplicities

x-intercepts: Set $y = 0$

$0 = x^4 + 2x^3 - 24x^2$

$0 = x^2(x^2 + 2x - 24)$

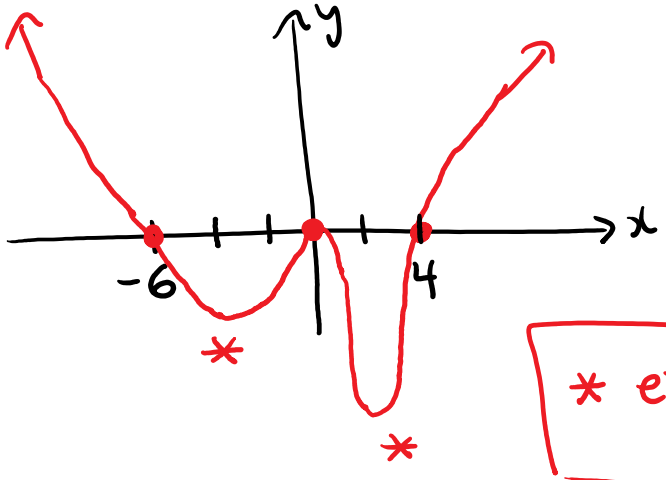
$0 = x^2(x+6)(x-4)$

$x = 0$ multiplicity 2 touches x-axis

$x = -6$ multiplicity 1 crosses

$x = 4$ 1 "

3) Graph



* exact points unknown

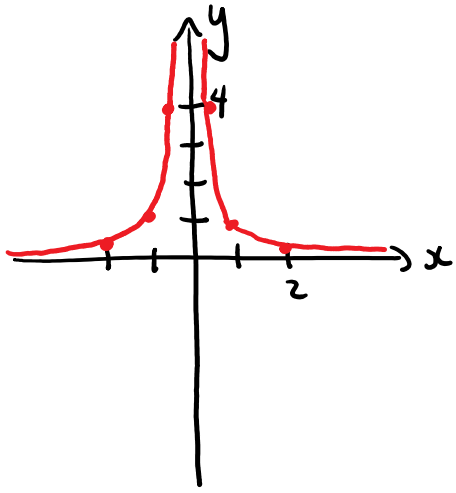
5.2 Properties of Rational Functions

rational function $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

Ex: Graph $f(x) = \frac{1}{(x-3)^2}$ using transformations

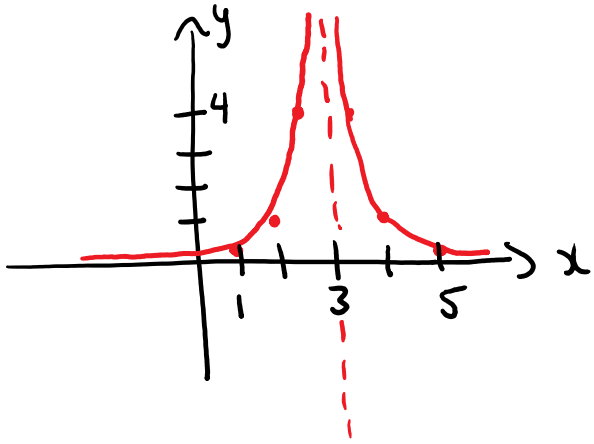
1) Graph $f(x) = \frac{1}{x^2}$

x	$y = \frac{1}{x^2}$
-2	$1/4$
-1	1
$-\frac{1}{2}$	$\frac{1}{(-\frac{1}{2})^2} = \frac{1}{(\frac{1}{4})} = 4$
0	undefined
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$



2) Shift <u>right</u> 3 units to get

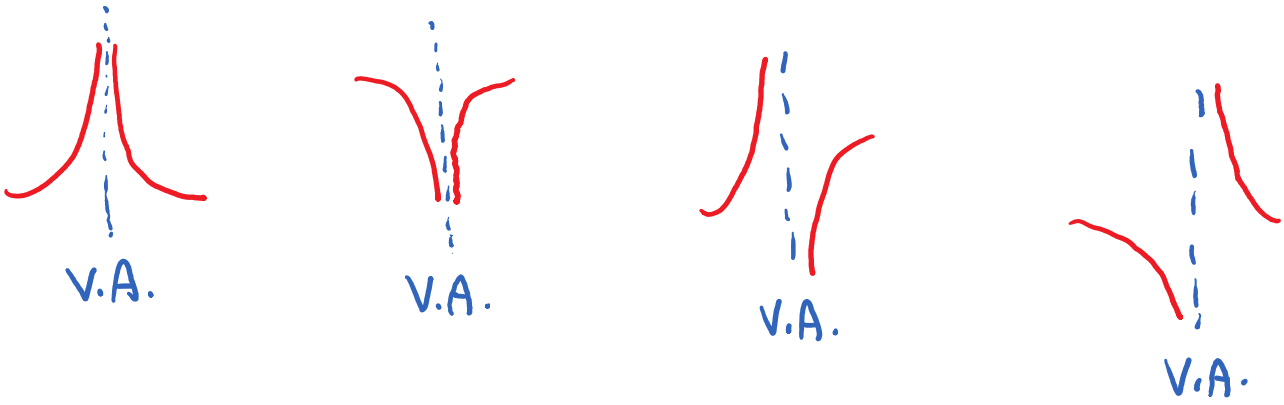
$$f(x) = \frac{1}{(x-3)^2}$$



DEFINITION

The vertical line $x=c$ is a <u>vertical asymptote</u> of $f(x)$ if $f(x) \to \pm\infty$ as $x \to c$

<u>Quick Ex of Vertical Asymptotes:</u>



V.A. $x = \#$

<u>Ex</u>: Refer to 2 graphs above

$f(x) = \frac{1}{x^2}$ has V.A. $x = 0$

$f(x) = \frac{1}{(x-3)^2}$ $x = 3$

To find V.A. : reduce $f(x)$ to lowest terms and find zeros of denominator

Ex: Find any V.A.

a) $f(x) = \dfrac{x+4}{x+5}$

$x+5=0$

$x=-5$

V.A. $x=-5$

b) $f(x) = \dfrac{x+3}{x^2+7x+12}$

$= \dfrac{(x+3)(1)}{(x+3)(x+4)}$

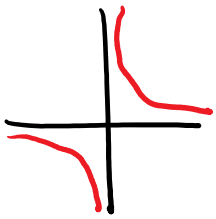
$= \dfrac{1}{x+4} \quad (x \neq -3)$

V.A. $x+4=0$

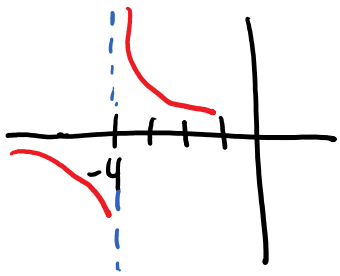
$x=-4$

DISCUSSION

$y = \frac{1}{x}$



$y = \frac{1}{x+4}$



$y = \frac{1}{x+4} \quad (x \neq -3)$