

Quiz Tues 3.3

Omit Section 3.3 # 69, 71, 73

3.3 Properties of Functions

- Symmetry
- Maximum/minimum points

Average rate of change

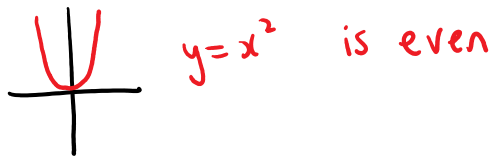
Symmetry

Even function: $f(-x) = f(x)$

Symmetric about y-axis

Ex: $f(x) = x^2$ is even

$$f(-x) = (-x)^2 = x^2 = f(x)$$

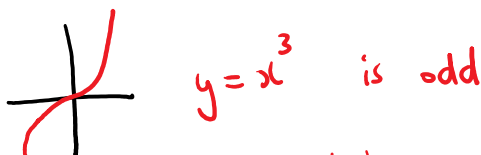


Odd function: $f(-x) = -f(x)$

Symmetric about origin

Ex: $f(x) = x^3$ is odd

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$



(Unchanged when we rotate by 180°)

Ex: Even, odd or neither?

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Check algebraically.

a) $f(x) = x^2 + 2x$

$$\begin{aligned} f(-x) &= (-x)^2 + 2(-x) \\ &= x^2 - 2x \end{aligned}$$

NEITHER

b) $f(x) = x^3 + 6x$

$$\begin{aligned} f(-x) &= (-x)^3 + 6(-x) \\ &= -x^3 - 6x \end{aligned}$$

$$= -(x^3 + 6x)$$

$$= -f(x)$$

ODD

c) $f(x) = |x|$

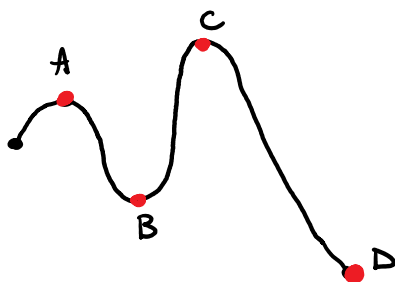
$$f(-x) = |-x|$$

$$= |x|$$

$$= f(x)$$

EVEN

Maximum / Minimum Points



Local maximum point (hilltop) : A, C

Absolute maximum point (the highest) : C

Local minimum point (valley floor) : B

Absolute minimum point (the lowest) : D

Absolute minimum point (the lowest) : D

Note: An endpoint can be an absolute max/min but not a local max/min.

Terminology :

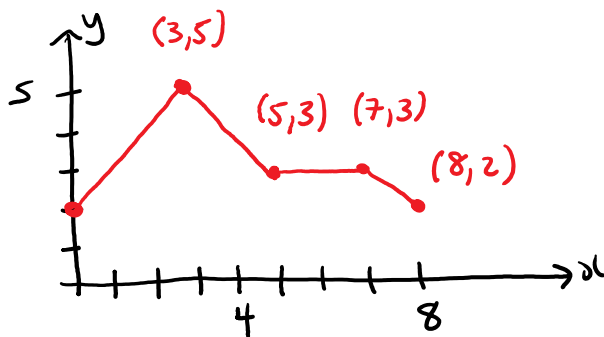
$\cup$
(2,3)

$x=2, y=3$

The function has an absolute (and local) minimum of 3 at $x=2$

y

Ex:



a) Absolute max ?

Absolute max of 5 at $x=3$

b) Absolute min ?

Absolute min of 2 at $x=0, 8$

c) Local max ? (Hilltops)

Local max of 5 at $x=3$

d) Local min ? (Valley floors)

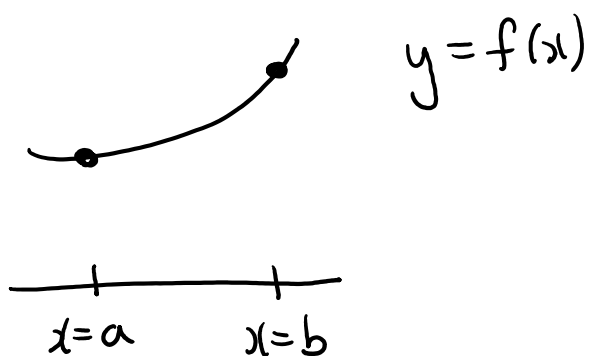
None

e) Where is f increasing? (x-values)
from $x=0$ to $x=3$ ✓
 $0 < x < 3$ ✓
as interval: $(0, 3)$ ✓

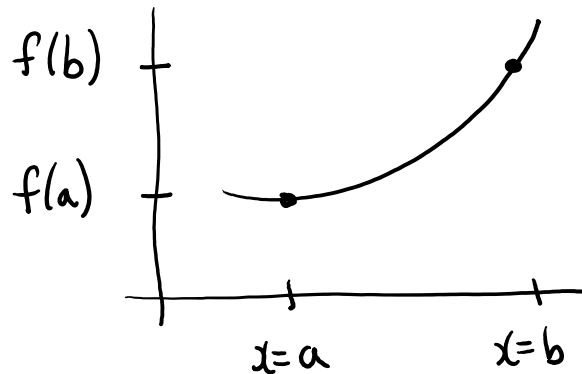
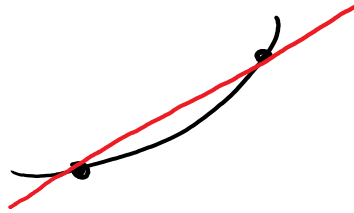
f) Where is f constant?
 $5 < x < 7$

g) Where is f decreasing?
 $3 < x < 5$ and $7 < x < 8$

Average Rate of Change



Definition: Average rate of change of f from $x=a$ to $x=b$ is the slope of the **line**:



$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

