

13.1 Sequences and Series Cont'd

Fun Fact

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

e.g. $1 + 2 + \dots + 7 = \frac{7(8)}{2}$

Using summation notation:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Ex: Calculate

a) $\sum_{k=1}^{20} k$

$$= 1 + 2 + 3 + \dots + 20$$

$$= \frac{20(21)}{2}$$

$$= 210$$

b) $\sum_{k=128}^{300} k$

$$= 128 + 129 + \dots + 300$$

$$= \underbrace{1 + 2 + \dots + 300} - \underbrace{(1 + 2 + \dots + 127)}$$

$$= \frac{300(301)}{2} - \frac{127(128)}{2}$$

$$= 37,022$$

$$c) \sum_{k=1}^{50} (3k+4)$$

$$= \sum_{k=1}^{50} 3k + \sum_{k=1}^{50} 4$$

$$= 3 \left(\sum_{k=1}^{50} k \right) + \left(\sum_{k=1}^{50} 4 \right)$$

Formula Logic

$$= 3 \cdot \frac{50(51)}{2} + \underbrace{(4+4+\dots+4)}_{50 \text{ terms}}$$

$$= 3 \cdot \frac{50(51)}{2} + 50(4)$$

$$= 4025$$

$$d) \sum_{k=1}^{70} (2k-3)$$

$$= \sum_{k=1}^{70} 2k - \sum_{k=1}^{70} 3$$

$$= 2 \sum_{k=1}^{70} k - \sum_{k=1}^{70} 3$$

Formula Logic

$$= 2 \cdot \frac{70(71)}{2} - 3(70)$$

$$= 4760$$

Announcements

- Exam Wed 24th 1:30pm TEC 175
- Music / earplugs
- Office Hours CBA 151
Tues 16th / Wed 17th / Wed 24th 11:30-1:00
- Exam Breakdown (Monday's notes)
- Review Problems

Omit #18

Full Solutions www.leahhoward.com

These review problems are not comprehensive.
We'll do 9 additional problems in class.

Additional Review Problems

① Solve $\frac{x+1}{x+2} = \frac{3}{4}$

$$4(x+1) = 3(x+2)$$

$$4x+4 = 3x+6$$

$$x = 2$$

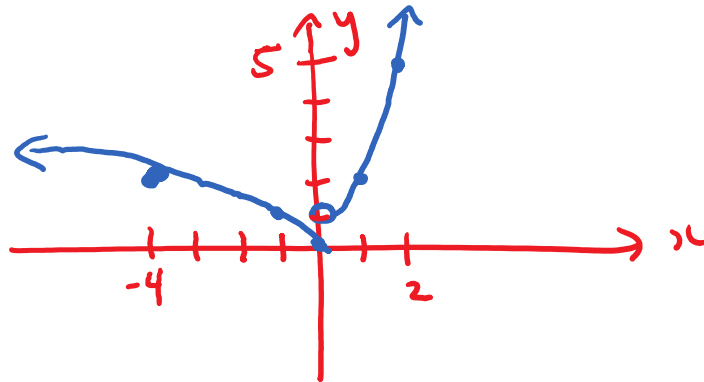
Check $x=2$: $LS = \frac{3}{4}$ $RS = \frac{3}{4}$ ✓

$$\boxed{x=2}$$

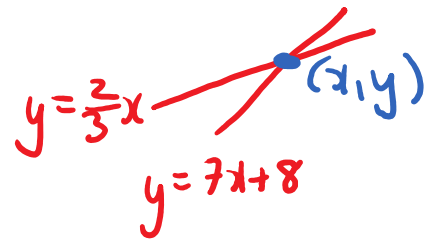
② Graph $f(x) = \begin{cases} \sqrt{-x} & \text{if } x \leq 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$

	x	y	
$x \leq 0$	$\left\{ \begin{array}{l} -4 \\ -1 \\ 0 \end{array} \right\}$	$\left\{ \begin{array}{l} 2 \\ 1 \\ 0 \end{array} \right\}$	$y = \sqrt{-x}$
$x > 0$	$\left\{ \begin{array}{l} 0 \\ 1 \\ 2 \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 2 \\ 5 \end{array} \right\}$	$y = x^2+1$

closed circle ($x \leq 0$)
open circle ($x > 0$)



- ③ Find the point where $f(x) = 7x + 8$ and $g(x) = \frac{2}{3}x$ intersect. Give exact values.



$$y = y$$

$$\frac{2}{3}x = 7x + 8$$

$$2x = 21x + 24$$

$$2x - 21x = 24$$

$$-19x = 24$$

$$x = -\frac{24}{19}$$

Plug into either

$$y = \frac{2}{3}x$$

$$y = \frac{2}{3} \left(\frac{-24}{19} \right)$$

$$y = \frac{-16}{19}$$

$$\left(\frac{-24}{19}, \frac{-16}{19} \right)$$