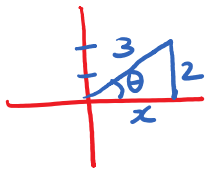


Find $\tan(\arcsin \frac{2}{3})$

$$\theta = \arcsin \frac{2}{3}$$

$$\sin \theta = \frac{2}{3}$$

$$\frac{y}{r} = \frac{2}{3}$$



$$x^2 + 4 = 9$$

$$x^2 = 5$$

$$x = \pm \sqrt{5}$$

$$x = \sqrt{5}$$

$$\text{Want } \tan \theta = \frac{y}{x}$$

$$= \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

13.1 Sequences and Series Cont'd

Series: a **sum**, particularly when terms have a pattern

$$\text{e.g. } 2 + 4 + 6 + 8 + \dots + 80$$

"Summation Notation" or "Sigma Notation"

Ex: Expand

$$a) \sum_{k=1}^4 \frac{1}{k}$$

means: the sum of $\frac{1}{k}$
where k ranges from 1 to 4

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$(k=1) \quad (k=2) \quad (k=3) \quad (k=4)$

$$b) \sum_{k=2}^4 \frac{k+1}{4k}$$

$$= \frac{3}{8} + \frac{4}{12} + \frac{5}{16}$$

$(k=2) \quad (k=3) \quad (k=4)$

Ex: Calculate

$$a) \sum_{k=2}^5 (3k+1)$$

$$= 7 + 10 + 13 + 16$$

$(k=2) \quad (k=3) \quad (k=4) \quad (k=5)$

$$= 46$$

$$b) \sum_{k=1}^{10} (-1)^k$$

$$\begin{aligned}
 &= \underbrace{(-1)}_{(k=1)} + \underbrace{1}_{(k=2)} + \underbrace{(-1)}_{(k=3)} + \dots + \underbrace{1}_{(k=10)} \\
 &= 0
 \end{aligned}$$

c) $\sum_{k=1}^n a$ $a: \text{constant}$

$$\begin{aligned}
 &= a + a + a + \dots + a \\
 &\quad (k=1) \quad (k=2) \qquad \qquad \qquad (k=n) \\
 &= a(n)
 \end{aligned}$$

Ex: Write in summation notation (sigma notation)

$$\begin{aligned}
 \text{a) } &1 + 8 + 27 + 64 + \dots + 1000 \\
 &= 1^3 + 2^3 + 3^3 + \dots + 10^3 \\
 &= \sum_{k=1}^{10} k^3
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } &\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{64} \\
 &= \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^6}
 \end{aligned}$$

$$= \sum_{k=2}^6 \frac{1}{2^k}$$

Alternatively: $\sum_{k=2}^6 \left(\frac{1}{2}\right)^k$ or $\sum_{k=1}^5 \left(\frac{1}{2}\right)^{k+1}$

Properties of Summation Notation:

$$1) \sum_{k=1}^n c a_k = c \sum_{k=1}^n a_k$$

c : Constant

Ex: $\sum_{k=1}^{20} a_k = 30$ Find $\sum_{k=1}^{20} 4a_k$

$$= 4 \left(\sum_{k=1}^{20} a_k \right)$$

$$= 4(30)$$

$$= 120$$

$$2) \sum_{k=1}^n (a_k \pm b_k) = \left(\sum_{k=1}^n a_k \right) \pm \left(\sum_{k=1}^n b_k \right)$$

Ex: Given $\sum_{k=1}^{12} a_k = 50$

and $\sum_{k=1}^{12} b_k = 70$

Find $\sum_{k=1}^{12} (a_k - b_k)$

$$= \sum_{k=1}^{12} a_k - \sum_{k=1}^{12} b_k$$

$$= 50 - 70$$

$$= -20$$

Fun
Fact

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

$$\text{e.g. } 1 + 2 + 3 + \dots + 10 = \frac{10(11)}{2} \checkmark$$

$$\text{e.g. } 1 + \dots + 1000 = \frac{1000(1001)}{2} = 500,500$$