Week 13 Friday

April 5, 2019 7:45 AM

8.5 Sum and Difference Formulas Cont'd

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

Condensed Version:

$$\boxed{\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}}$$

Will be on formula sheet

Updated formula sheet on Monday

Ex: Find the exact value of

$\tan\frac{5\pi}{12}$

$$\boxed{\begin{aligned}\frac{5\pi}{12} &= \frac{2\pi}{12}+\frac{3\pi}{12}\\ &= \frac{\pi}{6}+\frac{\pi}{4}\end{aligned}}$$

$$= \tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1-(\tan\frac{\pi}{6})(\tan\frac{\pi}{4})}$$

$$= \frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}$$

✓ Mult by $\frac{3}{3}$:

$$= \frac{\sqrt{3}+3}{3-\sqrt{3}}$$

✓ Could rationalize

Ex: Let $0 < \alpha, \beta < \frac{\pi}{2}$

Given $\cos\alpha = \frac{\sqrt{3}}{2}$ and $\sin\beta = \frac{1}{4}$

Find $\sin(\alpha+\beta)$

Recall $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Find $\sin\alpha$: $\sin^2\alpha + \cos^2\alpha = 1$

$$\sin^2\alpha + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$$

$$\sin^2\alpha + \frac{3}{4} = 1$$

$$\sin^2\alpha = \frac{1}{4}$$

$$\sin\alpha = \pm\frac{1}{2}$$

S A
T C

$$\sin\alpha = \frac{1}{2}$$

Find $\cos\beta$: $\sin^2\beta + \cos^2\beta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\beta = 1$$

$$\cos^2\beta = \cancel{1 - \frac{1}{16}} \quad \frac{15}{16}$$

$$\cos\beta = \pm\frac{\sqrt{15}}{4}$$

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$$\cos\beta = \frac{\sqrt{15}}{4}$$

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$$\boxed{\cos\alpha = \frac{\sqrt{3}}{2} \qquad \sin\beta = \frac{1}{4}}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{1}{2}\cdot\frac{\sqrt{15}}{4} + \frac{\sqrt{3}}{2}\cdot\frac{1}{4}$$

$$= \frac{\sqrt{15}+\sqrt{3}}{8}$$

8.6 Double Angle Formulas

Special case of 8.5

Use $\alpha = \theta$ $\beta = \theta$ $\rightarrow$ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\sin 2\theta = \underbrace{\sin\theta\cos\theta + \cos\theta\sin\theta}$$

$$= 2\sin\theta\cos\theta$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

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Memorize or know how to derive them

e.g.
$$\begin{aligned}\cos 2\theta &= \cos(\theta+\theta)\\ &= \cos\theta\cos\theta - \sin\theta\sin\theta\\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

Ex: Show that $\cos 2\theta$ can be rewritten as $1-2\sin^2\theta$ or $2\cos^2\theta - 1$

a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\boxed{\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \cos^2\theta &= 1-\sin^2\theta\end{aligned}}$$

$$\begin{aligned}&= (1-\sin^2\theta) - \sin^2\theta\\ &= 1 - 2\sin^2\theta\end{aligned}$$

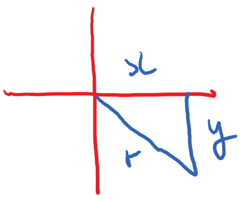
b)
$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= \cos^2\theta - (1-\cos^2\theta)\\ &= \cos^2\theta - 1 + \cos^2\theta\\ &= 2\cos^2\theta - 1\end{aligned}$$

Summary

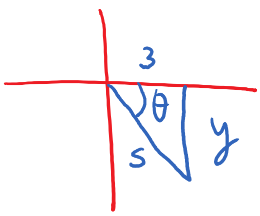
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

Ex: Find $\tan 2\theta$ if $\cos\theta = \frac{3}{5}$ and $\sin\theta < 0$

Find $\tan\theta$ (then use $\tan 2\theta$ formula)



$$\cos\theta = \frac{3}{5}$$
$$\frac{x}{r} = \frac{3}{5}$$



$$3^2 + y^2 = 5^2$$
$$y^2 = \cancel{5^2 - 3^2}\ 16$$
$$y = \pm 4$$

Use $y = -4$ (QIV)

$$\tan\theta = \frac{y}{x} = \frac{-4}{3}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{-\frac{8}{3}}{1 - \frac{16}{9}} \qquad \text{Multiply by } \frac{9}{9}$$

$$= \frac{-\frac{8}{3}(9)}{9 - 16}$$

$$= \frac{-24}{-7} \quad \text{or} \quad \frac{24}{7}$$