Week 13 Wednesday

April 3, 2019 7:41 AM

Test tomorrow
6.3 - 6.6, 6.8, 7.1 - 7.8
Sugg HW and Practice Problems

8.4 Trig Identities Cont'd

Ex: Show that $\frac{\csc\theta}{\cot\theta} = \sec\theta$

$$\frac{\csc\theta}{\cot\theta} = \frac{\left(\frac{1}{\sin\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}$$

Rewrite in terms of $\sin\theta$ and $\cos\theta$

$$= \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta \checkmark$$

Ex: Show that $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\csc\theta$

$$\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = \frac{(1-\cos\theta)}{\sin\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)} + \frac{\sin\theta}{(1-\cos\theta)}\cdot\frac{(\sin\theta)}{(\sin\theta)}$$

Common denominator

$$= \frac{(1-\cos\theta)^2 + \sin^2\theta}{\sin\theta(1-\cos\theta)} \leftarrow \text{expand}$$

$$= \frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)} \quad (\cos^2\theta + \sin^2\theta = 1)$$

$$= \frac{2 - 2\cos\theta}{\sin\theta(1-\cos\theta)} \leftarrow \text{factor}$$

$$= \frac{2(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\csc\theta \checkmark$$

Ex: Prove that $\sec\theta - \tan\theta = \dfrac{\cos\theta}{1+\sin\theta}$

$$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{(1+\sin\theta)} \cdot \frac{(1-\sin\theta)}{(1-\sin\theta)}$$

Multiply top and bottom by $1-\sin\theta$
"the conjugate of $1+\sin\theta$"

$$= \frac{\cos\theta\,(1-\sin\theta)}{1-\sin^2\theta} \leftarrow$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1-\sin^2\theta$

$$= \frac{\cos\theta\,(1-\sin\theta)}{\cos^2\theta\ \cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta \checkmark$$

Techniques

1. Factor
2. Rewrite in terms of $\sin\theta$ and $\cos\theta$
3. Common denominator
4. Multiply by conjugate

If you see $1 \pm \cos\theta$, multiply top and bottom by $1 \mp \cos\theta$

$1 + \sin\theta$ $\quad$ $1 - \sin\theta$

$1 - \sin\theta$ $\quad$ $1 + \sin\theta$

8.5 Sum and Difference Formulas

Formula Sheet:

$A = Pe^{rt}$ (6.8)

~~$A = P\left(1 + \frac{r}{n}\right)^{nt}$ (6.7)~~

Two trig formulas (8.5)

$s = r\theta$ (Ch 7, 7.1?)

Last two (Ch 13)

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

α, β: any angle

$\cos(10° + 35°) = \cos 10°\cos 35° - \sin 10°\sin 35°$

Ex: Find exact values

a) $\cos 15°$

$15° = 60° - 45°$

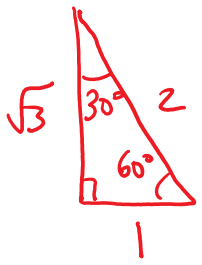
$$= \cos(60^\circ - 45^\circ)$$

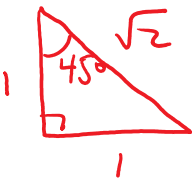
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2}\,\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$





$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $\sin 75^\circ \cos 15^\circ - \cos 75^\circ \sin 15^\circ$

$\alpha = 75^\circ$
$\beta = 15^\circ$

$= \sin\alpha \cos\beta - \cos\alpha \sin\beta$

$= \sin(\alpha - \beta)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$