Week 12 Friday

March 29, 2019 7:45 AM

8.1 Inverse Trig Functions Cont'd

Recap:

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\sin^{-1} -\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$

arcsin x outputs angles in QI and QIV

arctan x " QI and QIV

arccos x " QI and QII

S	A
T	C

Composing Functions

$\tan(\tan^{-1} x) = x$ when $-\infty < x < \infty$

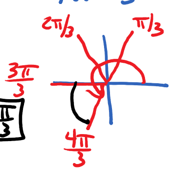
$\tan^{-1}(\tan x) = x$ when x is in QI or QIV $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

Ex: a) $\tan^{-1}\left(\tan \frac{-\pi}{4}\right)$

$= -\frac{\pi}{4}$ (in QI or QIV ✓)

b) $\tan^{-1}\left(\underbrace{\tan \frac{4\pi}{3}}\right)$

$\tan \frac{4\pi}{3} = \sqrt{3}$



$\boxed{\alpha = \frac{\pi}{3}}$

S	A
T	C

$\tan \frac{4\pi}{3} = \tan \frac{\pi}{3} = \sqrt{3}$

$= \tan^{-1} \sqrt{3}$

$= \frac{\pi}{3}$

$\cos(\cos^{-1} x) = x$ when $-1 \le x \le 1$

$\cos^{-1}(\cos x) = x$ when x is in QI or QII $(0 \le x \le \pi)$

Ex: a) $\cos^{-1}\left(\cos \frac{\pi}{6}\right)$

$= \frac{\pi}{6}$ (in QI or QII ✓)

b) $\cos^{-1}\left(\underbrace{\cos \frac{-\pi}{3}}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$

Ex: Solve $6 \arccos 3x = 5\pi$

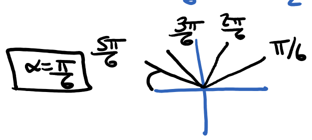
$\arccos 3x = \frac{5\pi}{6}$

Apply cos function:

$\cos(\arccos 3x) = \cos \frac{5\pi}{6}$

$3x = \cos \frac{5\pi}{6}$

$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$



S	A
T	C

$\cos \frac{5\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

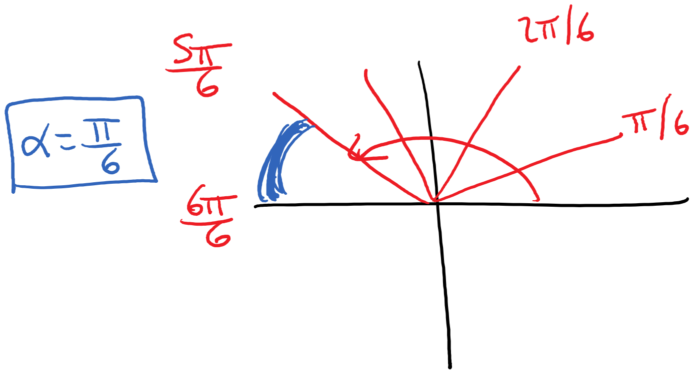
Reference Angle

CAST Rule (+/-)

$3x = -\frac{\sqrt{3}}{2}$

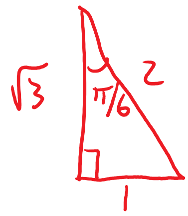
$x = -\frac{\sqrt{3}}{6}$

$\tan \frac{5\pi}{6}$



S	A
T	C

$$\tan\frac{5\pi}{6} = -\tan\frac{\pi}{6}$$



$$= -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

8.2 Inverse Trig Cont'd

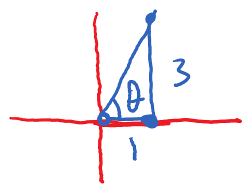
Recall $\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

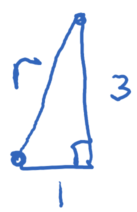
Ex: Find $\sin(\tan^{-1} 3)$

Let $\theta = \tan^{-1} 3$

$$\tan\theta = 3$$

$$\frac{y}{x} = \frac{3}{1}$$



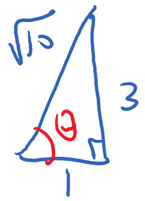


$$1^2 + 3^2 = r^2$$

$$10 = r^2$$

$$\pm\sqrt{10} = r$$

$r = \sqrt{10}$



Want $\sin\theta$

$$= \frac{y}{r}$$

$$= \frac{3}{\sqrt{10}} \quad \text{or} \quad \frac{3\sqrt{10}}{10}$$