

8.1 Inverse Trig Functions

$\tan^{-1} x$ or $\arctan x$ is the inverse tangent function

NOT TO BE CONFUSED WITH

$$\cot x = \frac{1}{\tan x} = (\tan x)^{-1}$$

Ex: $\tan \frac{\pi}{4} = 1$

$$\arctan 1 = \frac{\pi}{4}$$

$$\tan(\text{angle}) = \#$$

$$\arctan(\#) = \text{angle}$$

FACT

$$\tan^{-1}(-a) = -\tan^{-1}a$$

Ex: $\tan^{-1}(-\sqrt{3}) = -\tan^{-1}\sqrt{3}$

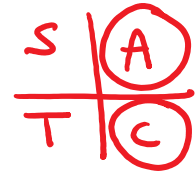
$$= -\frac{\pi}{3}$$



FACT

Domain of $\arctan x$: $-\infty < x < \infty$

Range of $\arctan x$: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$



The function $y = \arctan x$
returns angles in QI or QIV

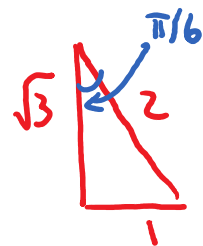
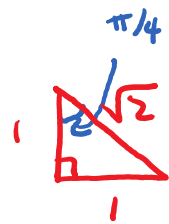
$\cos^{-1} x$ or $\arccos x$ is the
inverse cosine function

NOT TO BE CONFUSED WITH

$$(\cos x)^{-1} = \frac{1}{\cos x} = \sec x$$

Ex: $\cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$$\cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

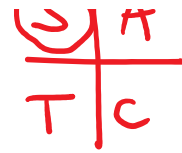


FACT

$$\cos^{-1}(-a) = \pi - \cos^{-1} a$$



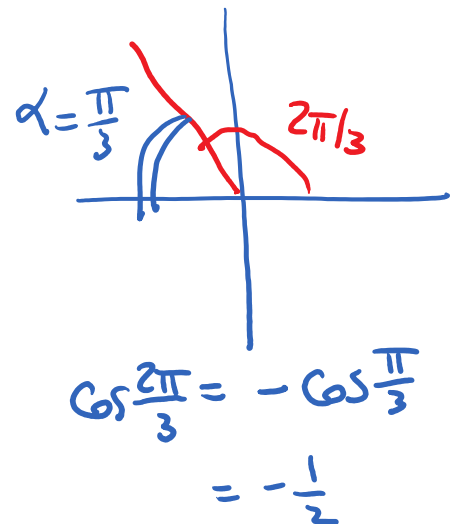
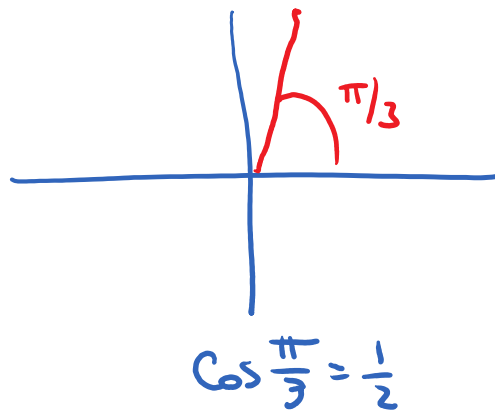
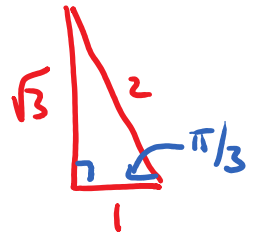
$$\boxed{\cos^{-1}(-a) = \pi - \cos^{-1}a}$$



Ex: $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$



FACT REVISITED

$$\cos^{-1}(-a) = \pi - \cos^{-1}a$$

$$= \pi - \text{Reference Angle}$$

FACT

Domain of $\arccos x$:

$$-1 \leq x \leq 1$$

Range of $\arccos x$:

$$0 \leq \arccos x \leq \pi$$

(QI or QII angles)

(QI or QII angles)

Ex: Find:

$$\begin{aligned} \text{a) } \sin^{-1}\left(-\frac{3}{4}\right) \\ = -0.85 \text{ rads} \end{aligned}$$

$$\begin{aligned} \text{b) } \tan^{-1}(10,000) \\ \approx 1.57 \text{ rads} \quad (\text{close to } \frac{\pi}{2}) \end{aligned}$$

$$\begin{aligned} \text{c) } \sin^{-1}(-1) \\ = -\sin^{-1} 1 \\ = -\frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} \sin \frac{\pi}{2} &= 1 \\ \sin^{-1} 1 &= \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } \cos^{-1}(-1) \\ = \pi \end{aligned}$$

$$\begin{aligned} \cos \pi &= -1 \\ \cos^{-1}(-1) &= \pi \end{aligned}$$

$$\begin{aligned} \text{Alternatively: } \cos^{-1}(-1) &= \pi - \cos^{-1} 1 \\ &= \pi - 0 \\ &= \pi \end{aligned}$$

Facts

1) $\sin(\sin^{-1} x) = x$ when $-1 \leq x \leq 1$

2) $\sin^{-1}(\sin x) = x$ when x is in QI or QIV
($-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$)

Ex: a) $\sin(\sin^{-1} 0.2) = 0.2$

b) $\sin^{-1}(\sin \frac{\pi}{12}) = \frac{\pi}{12}$

c) $\sin^{-1}(\sin \frac{5\pi}{6}) = \sin^{-1}(\frac{1}{2})$
CAUTION
 $= \frac{\pi}{6}$

