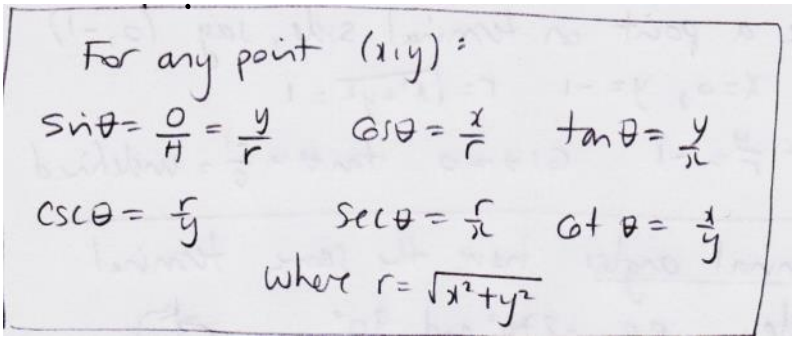
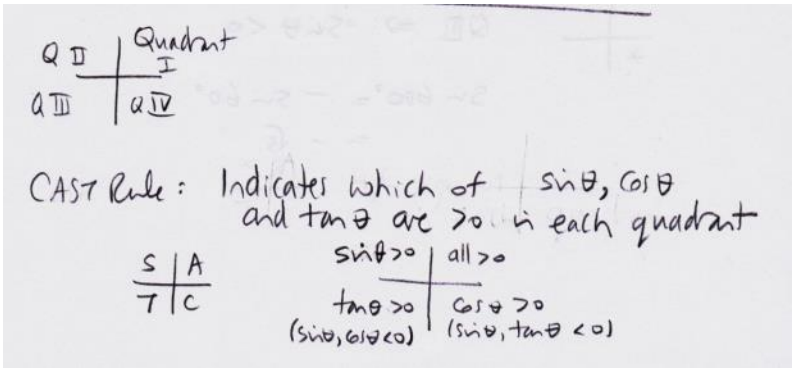
Week 11 Wednesday

March 20, 2019 7:43 AM

7.4 Cont'd





Ex: Given $\cos\theta = -\frac{1}{4}$ and $\sin\theta < 0$
find the other 5 values.

$\cos\theta, \sin\theta < 0 \Rightarrow$ QIII

$\cos\theta = \frac{x}{r} = \frac{-1}{4}$

$\boxed{x=-1}$ $\boxed{r=4}$ $y = ?$

$r = \sqrt{x^2+y^2}$

$4 = \sqrt{1+y^2}$

$16 = 1+y^2$

$15 = y^2$

$y = \pm\sqrt{15}$

QIII $\Rightarrow y<0$ $\boxed{y = -\sqrt{15}}$

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{15}}{4}$ $\cos\theta = \frac{-1}{4}$ $\tan\theta = \frac{y}{x} = \sqrt{15}$

$\csc\theta = \frac{-4}{\sqrt{15}} = \frac{-4\sqrt{15}}{15}$ $\sec\theta = -4$ $\cot\theta = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

7.5 Properties of Trig Functions

1) Domain
2) Periodic Behaviour
3) Odd-Even (symmetry)

Will be used for graphing in Sections 7.6/7.7

Function	Domain
$\tan\theta = \frac{y}{x}$, $\sec\theta = \frac{r}{x}$	$x \neq 0$ $\theta \neq 90°, 270°, \ldots$ $-90°, -270°, \ldots$ $\theta \neq$ odd multiples of $90°$
$\csc\theta = \frac{r}{y}$, $\cot\theta = \frac{x}{y}$	$y \neq 0$ $\theta \neq 0°, 180°, 360°, \ldots$ $-180°, \ldots$ $\theta \neq$ multiples of $180°$
$\sin\theta = y/r$, $\cos\theta = x/r$	$r \neq 0$ $-\infty < \theta < \infty$

r = distance > 0

Periodic Properties

Let k be an integer

$\sin(\theta + 2\pi k) = \sin\theta$

Ex: $\sin\frac{\pi}{2} = \sin\frac{5\pi}{2} = \sin\frac{9\pi}{2} = \sin\frac{-3\pi}{2}$

$+2\pi$, $+4\pi$, -2π

$\tan(\theta + \pi k) = \tan\theta$

Ex: $\tan\frac{\pi}{4} = \tan\frac{5\pi}{4} = \tan\frac{-3\pi}{4} = \tan\frac{-7\pi}{4}$

$+\pi$, $-\pi$, -2π

SUMMARY

Periodic Properties

Let k be an integer

$\sin(\theta + 2\pi k) = \sin\theta$
$\cos(\theta + 2\pi k) = \cos\theta$
$\csc(\theta + 2\pi k) = \csc\theta$
$\sec(\theta + 2\pi k) = \sec\theta$

"Period = 2π"

$\tan(\theta + \pi k) = \tan\theta$
$\cot(\theta + \pi k) = \cot\theta$

"Period = π"

Ex: Simplify

a) $\tan(x + 3\pi) = \tan x$ Period = π

b) $\sin(x + 3\pi) = \sin(x + \pi)$ Period = 2π

c) $\tan 210° = \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ Period = $180°$

d) $\sin 450° = \sin 90° = \frac{y}{r} = 1$ Period = $360°$