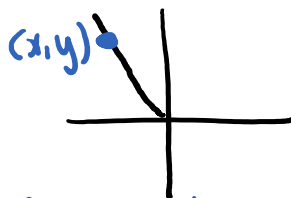


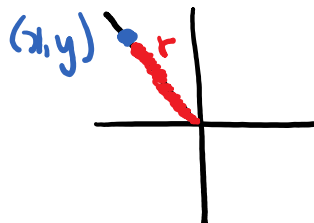
Quiz tomorrow 6.6

7.4 Trig Functions of any Angle

No longer restricted to $0^\circ < \theta < 90^\circ$



Any point on
the terminal side
of θ



distance from (x, y) to $(0, 0)$

$$r = \sqrt{x^2 + y^2}$$

For any point (x, y) :

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

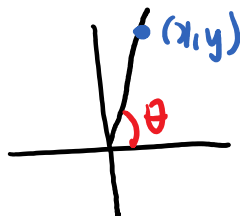
$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

$$\text{where } r = \sqrt{x^2 + y^2}$$



Comment: If θ is in Quadrant I ($0^\circ < \theta < 90^\circ$),
these formulas are the same as SOHCAHTOA



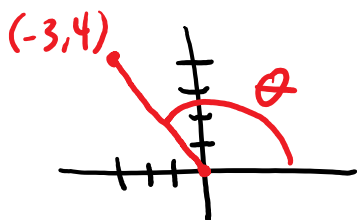
SOHCAHTOA

$$\cos \theta = \frac{A}{H} = \frac{x}{r}$$

SOHCAHTOA only applies when $0^\circ < \theta < 90^\circ$



Ex: Find the 6 trig functions of θ if $(-3, 4)$ is a point on the terminal side of θ .



$$x = -3$$

$$y = 4$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{-3}{5}$$

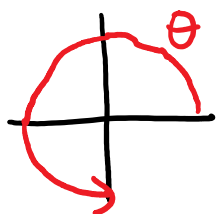
$$\tan \theta = \frac{y}{x} = \frac{4}{-3} \text{ or } -\frac{4}{3}$$

$$\csc \theta = \frac{r}{y} = \frac{5}{4}$$

$$\sec \theta = \frac{r}{x} = \frac{5}{-3} \text{ or } -\frac{5}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{-3}{4}$$

Ex: Find $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $\theta = 270^\circ$



Choose any point on the terminal side

Choose any point on the terminal side

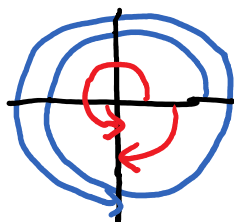
$$(x, y) = (0, -1)$$

$$x=0 \quad y=-1 \quad r = \sqrt{x^2 + y^2} \\ = \sqrt{1} \\ = 1$$

$$\sin \theta = \frac{y}{r} = -1 \quad \cos \theta = \frac{x}{r} = 0 \quad \tan \theta = \frac{y}{x} = \frac{-1}{0} \\ \text{undefined!}$$

DEF

Coterminal angles have the same terminal side



270° and -90° are coterminal
and 630°

FACT

Coterminal angles differ by a multiple of 360°
" " 2π rad

Ex: Find a coterminal angle θ so that
 $0^\circ \leq \theta < 360^\circ$ or $0 \leq \theta < 2\pi$

a) 2100°

2100° - 360° too big

2100° - 2(360°) "

⋮

$$2100^\circ - 4(360^\circ) \quad "$$

$$2100^\circ - 5(360^\circ) = 300^\circ$$

$$\theta = 300^\circ$$

b) $-\frac{14}{3}\pi$

$$-\frac{14}{3}\pi + 2\pi \text{ is negative}$$

$$-\frac{14}{3}\pi + 2(2\pi) = -\frac{14\pi}{3} + \frac{12\pi}{3} \text{ still negative}$$

$$-\frac{14}{3}\pi + 3(2\pi) = -\frac{14\pi}{3} + \frac{18\pi}{3} = \frac{4\pi}{3}$$

$$\theta = \frac{4\pi}{3}$$

FACT

Coterminal angles have the same trig values!

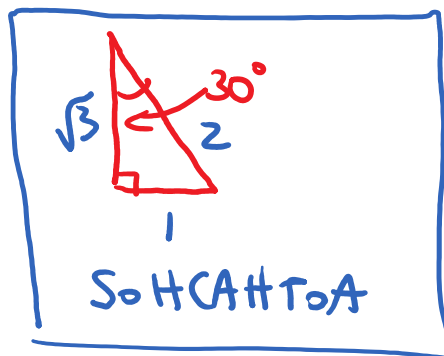
Ex: Find:

a) $\cos 390^\circ$

$$= \cos 30^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$390^\circ - 360^\circ = 30^\circ$$



$$b) \csc\left(-\frac{7\pi}{4}\right)$$

$$-\frac{7\pi}{4} + 2\pi = -\frac{7\pi}{4} + \frac{8\pi}{4} = \frac{\pi}{4}$$

$$= \csc\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sin}$$

$$= \frac{\sqrt{2}}{1}$$

$$= \sqrt{2}$$

