Week 10 Thursday

March 14, 2019 8:03 AM

7.2 Cont'd

Notation: $\cos^2\theta$ means $(\cos\theta)^2$

$\csc^2\theta$ " $(\csc\theta)^2$

If $\cos\theta = \frac{2}{3}$ then $\cos^2\theta = \frac{4}{9}$

FACT $\sin^2\theta + \cos^2\theta = 1$ for angle θ ★

★ $\div \sin^2\theta$: $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}\right)^2$

Recall $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$\frac{1}{\sin\theta} = \csc\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\div \cos^2\theta$: $\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$

$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$

Recall $\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Summary: 3 Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ ← Know this

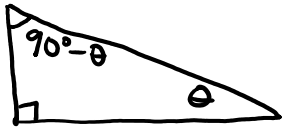
$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Memorize or be able to derive

FACT

In any triangle, angles sum to 180°



DEF

θ and 90°-θ are called Complementary angles

e.g. 20° and 70° are Complementary
45° and 45° "
8° and 82° "

FACT Complementary Angle Theorem

Cofunctions of complementary angles are equal

e.g. $\sin 35° = \cos 55°$

SINE AND COSINE ARE COFUNCTIONS
COTANGENT " TANGENT "
SECANT " COSECANT "

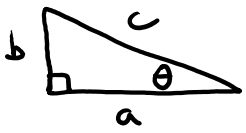
Quick Ex:

a) $\cot 15° = \tan 75°$

b) $\csc 80° = \sec 10°$

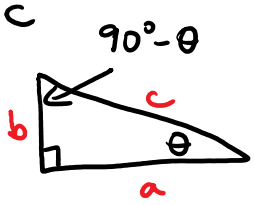
c) $\sin 25° = \cos 65°$

d) $\cos 11° = \sin 79°$

Ex: Given 

Confirm that $\sin\theta = \cos(90° - \theta)$

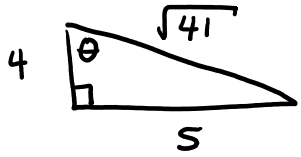
$$\sin\theta = \frac{O}{H} = \frac{b}{c}$$



$$\cos(90° - \theta) = \frac{A}{H} = \frac{b}{c}$$

Recap: 4 Big Ideas

1) SoHCAHToA



$\tan\theta = \frac{5}{4}$

2) Identities

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

3) Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

and 2 others

4) $\cot 19^\circ = \tan 71^\circ$

$\sin 85^\circ = \cos 5^\circ$

Ex: Simplify

a) $\cot 11^\circ - \dfrac{\cos 11^\circ}{\sin 11^\circ}$

$\dfrac{\cos\theta}{\sin\theta} = \cot\theta$

$= \cot 11^\circ - \cot 11^\circ$

$= 0$

b) $\sin^2\theta + \dfrac{1}{\sec^2\theta}$

$\dfrac{1}{\sec\theta} = \cos\theta$

$\dfrac{1}{\sec^2\theta} = \cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

c) $\sec^2 25^\circ - \tan^2 25^\circ$

$\sin^2\theta + \cos^2\theta = 1$

$\div \cos^2\theta$

$\dfrac{\sin^2\theta}{\cos^2\theta} + 1 = \dfrac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$$= (\tan^2 25^\circ + 1) - \tan^2 25^\circ$$

$$= 1$$