

R7. Rational Expressions

p.1

Ex: Reduce to lowest terms

$$\begin{aligned} \text{a) } \frac{x^3 + 27}{x^2 - 9} &= \frac{\cancel{(x+3)}(x^2 - 3x + 9)}{(x+3)\cancel{(x-3)}} \\ &= \frac{x^2 - 3x + 9}{x - 3} \end{aligned}$$

Restriction: $x \neq 3, -3$

$$\begin{aligned} \text{b) } \frac{6 - 2x}{x^2 + x - 12} &= \frac{2(3 - x)}{(x+4)(x-3)} \quad \left\{ \begin{array}{l} 3-x = -(x-3) \\ 3-x = -x+3 = -(x-3) \end{array} \right. \\ &= \frac{2(-1)(x-3)}{(x+4)\cancel{(x-3)}} \\ &= \frac{-2}{x+4} \end{aligned}$$

$x \neq -4, 3$

Multiplying and Dividing Fractions

p. 2

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad b, d \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad b, c, d \neq 0$$

Ex: Simplify

$$a) \quad \frac{x^2 - 6x + 9}{4x^2 + 12} \cdot \frac{4x^2 + 8}{x^2 + x - 12}$$

$$= \frac{(x-3)^2}{4(x^2+3)} \cdot \frac{4(x^2+2)}{(x+4)(x-3)}$$

$$= \frac{(x-3)(x^2+2)}{(x^2+3)(x+4)} \quad x \neq -4, 3$$

$$b) \quad \frac{\left(\frac{x+5}{x^2-4}\right)}{\left(\frac{x+1}{x^3-8}\right)} = \frac{x+5}{x^2-4} \cdot \frac{x^3-8}{x+1}$$
$$= \frac{x+5}{(x-2)(x+2)} \cdot \frac{(x-2)(x^2+2x+4)}{x+1}$$

$$= \frac{(x+5)(x^2+2x+4)}{(x+2)(x+1)}$$

$$x \neq 2, -2, -1$$

Adding / Subtracting: Same Denominator p.3

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b} \quad b \neq 0$$

Ex: Simplify

$$a) \quad \frac{x^2 - 4x}{x+6} + \frac{2x+16}{x+6}$$

$$= \frac{x^2 - 4x + 2x + 16}{x+6}$$

$$= \frac{x^2 - 2x + 16}{x+6} \leftarrow \text{prime} \quad x \neq -6$$

$$b) \quad \frac{x}{x-2} + \frac{5-x}{2-x}$$

Notice $-(2-x) = -2+x$
 $= x-2$

$$= \frac{x}{x-2} + \frac{(-1)(5-x)}{(-1)(2-x)}$$

$$= \frac{x}{x-2} + \frac{-5+x}{x-2} \leftarrow \text{same denominator}$$

$$= \frac{2x-5}{x-2} \quad x \neq 2$$

Adding/Subtracting : Different Denominators p.4

Ex: Simplify

$$a) \frac{x+5}{x^2-4} + \frac{x-7}{x^2-8x+12}$$

$$= \frac{x+5}{(x-2)(x+2)} + \frac{x-7}{(x-6)(x-2)}$$

$$\boxed{\text{LCM of denominators} = (x-2)(x+2)(x-6)}$$

$$= \frac{(x+5)(x-6)}{(x-2)(x+2)(x-6)} + \frac{(x-7)(x+2)}{(x-2)(x+2)(x-6)}$$

$$= \frac{x^2 - x - 30 + x^2 - 5x - 14}{(x-2)(x+2)(x-6)}$$

$$= \frac{2x^2 - 6x - 44}{(x-2)(x+2)(x-6)}$$

$$= \frac{2(x^2 - 3x - 22)}{(x-2)(x+2)(x-6)}$$

$$x \neq 2, -2, 6$$

$$b) \frac{1}{h} \left[\frac{1}{x+h} - \frac{1}{x} \right]$$

$$\swarrow \text{LCM} = (x+h)x$$

$$= \frac{1}{h} \left[\frac{x}{(x+h)x} - \frac{x+h}{x(x+h)} \right]$$

$$= \frac{1}{h} \left[\frac{x - x - h}{(x+h)x} \right]$$

$$= \frac{1}{h} \left[\frac{-h}{(x+h)x} \right]$$

$$= \frac{-1}{(x+h)x}$$

Ex: Complex Fractions

$$\frac{\frac{x}{x-2} + \frac{3}{x}}{\frac{1}{x} - \frac{x}{2}}$$

Look at all denominators:
 $x-2, x, 2$

$$\text{LCM} = 2x(x-2)$$

$$= \frac{\frac{x}{x-2} \cdot 2x(x-2) + \frac{3}{x} \cdot 2x(x-2)}{\frac{1}{x} \cdot 2x(x-2) - \frac{x}{2} \cdot 2x(x-2)}$$

Multiply all terms
 (top and bottom) by LCM

$$= \frac{2x^2 + 6(x-2)}{2(x-2) - x^2(x-2)}$$

7.9

p. 6

$$= \frac{2x^2 + 6x - 12}{2x - 4 - x^3 + 2x^2}$$

$$= \frac{2(x^2 + 3x - 6)}{-x^3 + 2x^2 + 2x - 4}$$

$$\left[\frac{1}{x} - \frac{1}{x+1} \right] \frac{1}{x}$$

$$\left[\frac{x+1}{(x+1)x} - \frac{x}{x(x+1)} \right] \frac{1}{x} =$$

$$\left[\frac{x+1-x}{x(x+1)} \right] \frac{1}{x} =$$

$$\left[\frac{1}{x(x+1)} \right] \frac{1}{x} =$$

$$\frac{1}{x^2(x+1)}$$

Ex: Complex Fractions

$$\frac{\frac{3}{x} + \frac{x}{5-x}}{\frac{x}{5} - \frac{1}{x}}$$

Look at all denominators:
 $x, 5, x-x$
 $(x-x)(5-x) = M(x)$

omit 1/0 p/q/1/2M
 M(x) pd (method has pd)

$$(5-x) \cdot \frac{3}{x} + (x-x) \cdot \frac{x}{5-x} = \frac{3(5-x)}{x}$$

$$(5-x) \cdot \frac{x}{5} - (5-x) \cdot \frac{1}{x} = \frac{x(5-x)}{5} - \frac{5-x}{x}$$

$$\frac{(5-x) \cdot 2 + x \cdot 5}{(5-x) \cdot x - (5-x) \cdot 5}$$