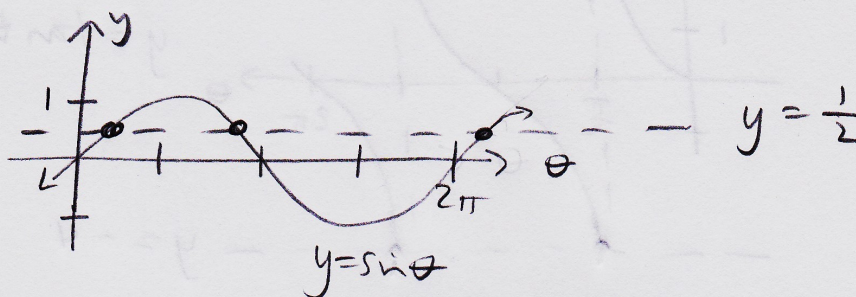


8.3 Trig Equations

p.1

Ex: Solve $\sin \theta = \frac{1}{2}$

1) Number of Solutions?



∞ -many solutions

2) Reference Angle

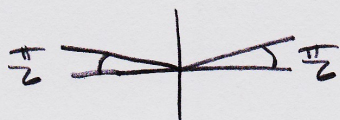
$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

3) CAST Rule

$$\sin \theta > 0$$

(S)	(A)
T	C



$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

4) Period

Period of $\sin \theta$ is 2π

We can add/subtract any multiple of 2π

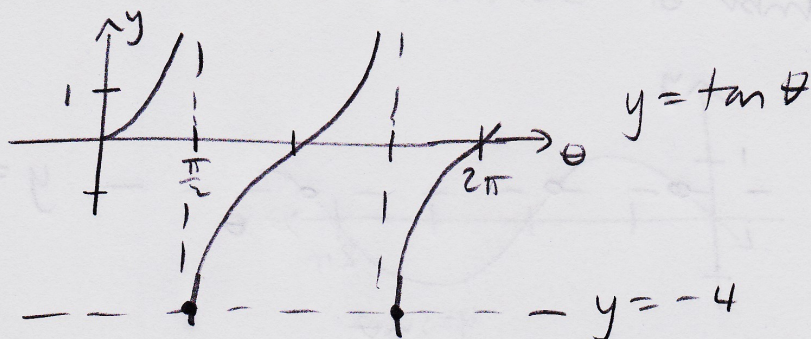
$$\theta = \frac{\pi}{6} + 2\pi k \quad \text{and} \quad \theta = \frac{5\pi}{6} + 2\pi k \quad k: \text{any integer}$$

Ex: Solve $\frac{1}{4}\tan\theta + 1 = 0$ on $0 \leq \theta < 2\pi$ p.2

$$\frac{1}{4}\tan\theta = -1$$

$$\tan\theta = -4$$

1) Number of solutions?

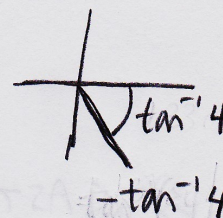


2 solutions

2) Reference Angle

$$\theta = \tan^{-1}(-4) = -\tan^{-1}4 \approx -1.33 \text{ rad}$$

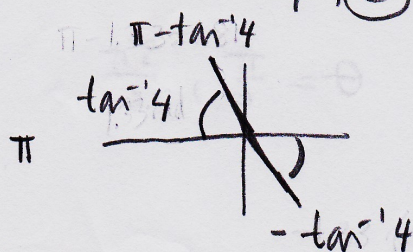
Reference Angle is $\tan^{-1}4 \approx 1.33 \text{ rad}$



3) CAST

$$\tan\theta < 0$$

(S)	A
T	(C)



$$\theta = -\tan^{-1}4, \pi - \tan^{-1}4$$

4) Period of $\tan\theta$ is π

$$\theta = -\tan^{-1}4, \pi - \tan^{-1}4, 2\pi - \tan^{-1}4$$

$\xrightarrow{+\pi}$ $\xrightarrow{+\pi}$

Want $0 \leq \theta < 2\pi$: $\boxed{\theta = \pi - \tan^{-1}4, 2\pi - \tan^{-1}4}$

Ex: Solve $\sin 2\theta = \frac{\sqrt{3}}{2}$ on $0 \leq \theta \leq 2\pi$

p.3

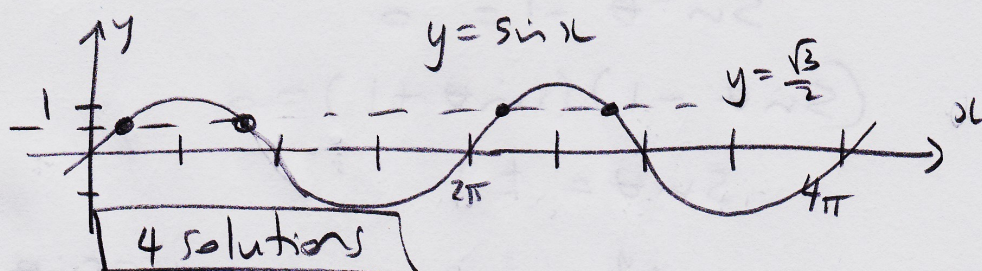
Let $2\theta = x$ $\sin x = \frac{\sqrt{3}}{2}$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq 2\theta \leq 4\pi$$

$0 \leq x \leq 4\pi$

1) Number of solutions?



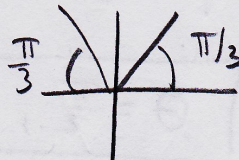
2) Reference angle

$$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$$

3) CAST

$$\sin x > 0$$

S	A
T	C



$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

4) Period of $\sin x$ is 2π

Want $0 \leq x \leq 4\pi$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3} = 2\theta$$

$\underbrace{\hspace{10em}}_{+2\pi}$

$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

Ex: Solve $2\sin^2\theta + \cos^2\theta - 2 = 0$
on $0 \leq \theta \leq 2\pi$

p.4

Recall $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \sin^2\theta + \cos^2\theta - 2 = 0$$

$$\sin^2\theta - 1 = 0$$

$$(\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\sin\theta = \pm 1$$



$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$