7.8 Phase Shift

Let $y = A\sin(wx - \phi) + C$ with $w > 0$

$y = A\sin\left[w\left(x - \frac{\phi}{w}\right)\right] + C$

has amplitude $= |A|$

period $= \frac{2\pi}{w}$

phase shift $= \frac{\phi}{w}$ units to the right

Similarly for cos, csc and sec functions.

$y = A\tan\left[w\left(x - \frac{\phi}{w}\right)\right] + C$ and

$y = A\cot\left[w\left(x - \frac{\phi}{w}\right)\right] + C$

have period $= \frac{\pi}{w}$ ←

phase shift $= \frac{\phi}{w}$ units to the right

amplitude is not meaningful

Ex: Graph $y = \sin\left(2x - \frac{\pi}{3}\right)$

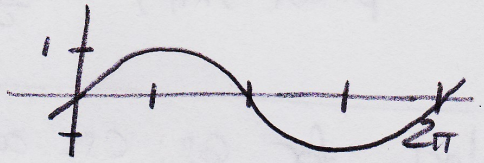
$y = \sin\left[2\left(x - \frac{\pi}{6}\right)\right]$

amplitude = 1

period = $\frac{2\pi}{2} = \pi$

phase shift = $\frac{\pi}{6}$ units right

x	$y = \sin x$
0	0
$\pi/2$	1
π	0
$3\pi/2$	−1
2π	0



New x-values:

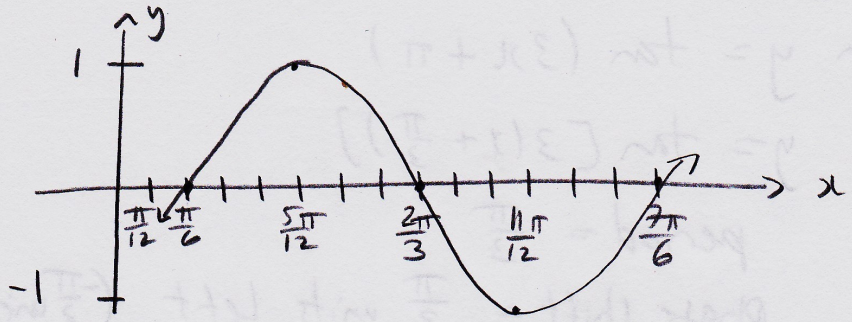
Divide each x by $w = 2$

Then add $\frac{\phi}{w} = \frac{\pi}{6}$

Caution: sign of $\frac{\phi}{w}$

$y = A\sin\left[w\left(x - \frac{\phi}{w}\right)\right] + C$

x			y
$0/2$	$+\frac{\pi}{6}$	$= \frac{\pi}{6}$	0
$\frac{\pi}{4}$	$+\frac{\pi}{6}$	$= \frac{5\pi}{12}$	1
$\frac{\pi}{2}$	$+\frac{\pi}{6}$	$= \frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	$+\frac{\pi}{6}$	$= \frac{11\pi}{12}$	−1
π	$+\frac{\pi}{6}$	$= \frac{7\pi}{6}$	0



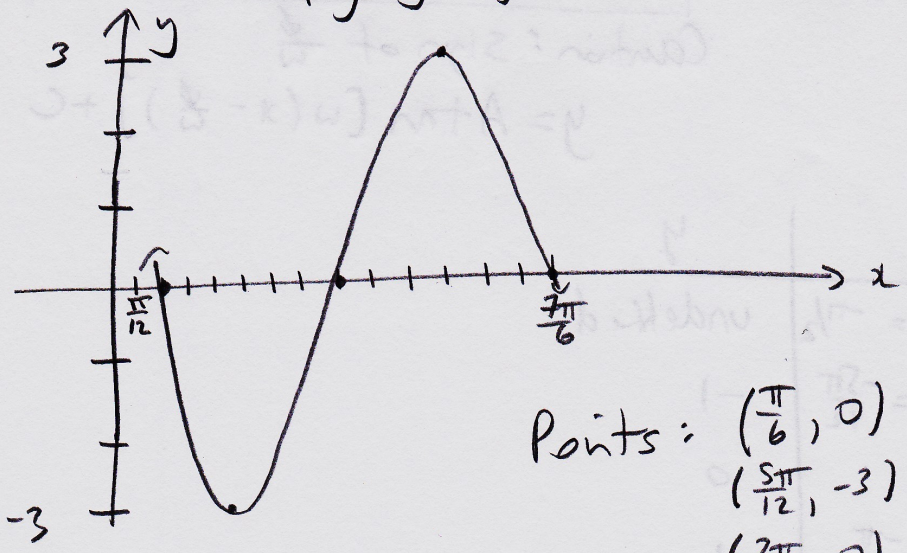
Ex: Graph $y = 3\sin\left(\frac{\pi}{3} - 2x\right)$

Need $w > 0$: $y = 3\sin\left[-\left(2x - \frac{\pi}{3}\right)\right]$

$y = -3\sin\left[2x - \frac{\pi}{3}\right]$ ($\sin\theta$ is odd)

$y = -3\sin\left[2\left(x - \frac{\pi}{6}\right)\right]$

Previous function with a vertical stretch and reflection (multiply y by -3)



Points: $\left(\frac{\pi}{6}, 0\right)$
$\left(\frac{5\pi}{12}, -3\right)$
$\left(\frac{2\pi}{3}, 0\right)$
$\left(\frac{11\pi}{12}, 3\right)$
$\left(\frac{7\pi}{6}, 0\right)$

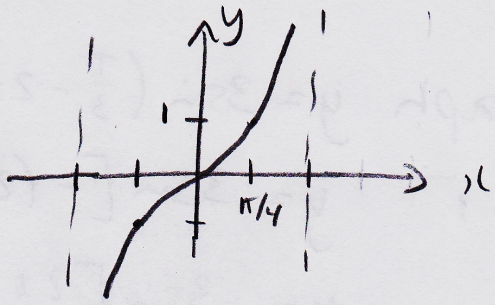
Ex: Graph $y = \tan(3x + \pi)$

$y = \tan[3(x + \frac{\pi}{3})]$

period $= \frac{\pi}{3}$

phase shift $= \frac{\pi}{3}$ units Left ($\frac{-\pi}{3}$ units right)

x	$y = \tan x$
$-\pi/2$	undefined
$-\pi/4$	-1
0	0
$\pi/4$	1
$\pi/2$	undefined



New x-values

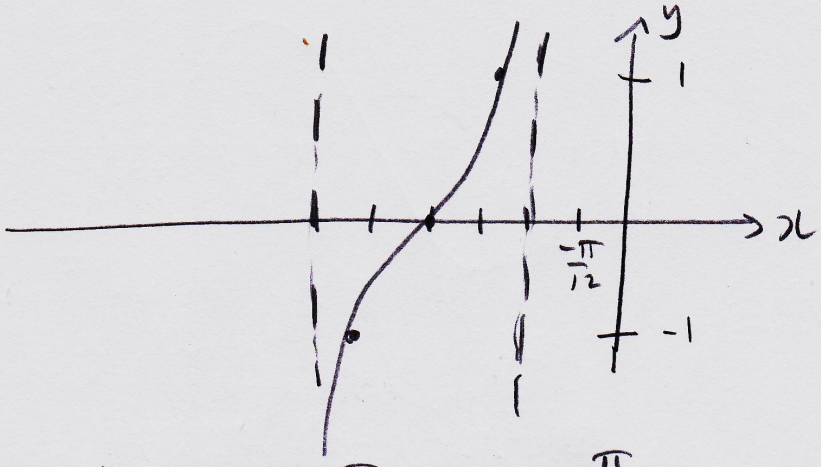
Divide each x by $w = 3$

Then add $\frac{\phi}{w} = \frac{-\pi}{3}$

Caution: Sign of $\frac{\phi}{w}$

$y = A\tan[w(x - \frac{\phi}{w})] + C$

x	y
$-\pi/6 - \pi/3 = -\pi/2$	undefined
$-\pi/12 - \pi/3 = \frac{-5\pi}{12}$	-1
$\frac{-\pi}{3}$	0
$\frac{\pi}{12} - \frac{\pi}{3} = \frac{-\pi}{4}$	1
$\frac{\pi}{6} - \frac{\pi}{3} = \frac{-\pi}{6}$	undefined



V.A. $x = \frac{-\pi}{2}$, $x = \frac{-\pi}{6}$