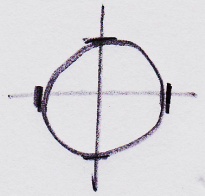
7.5 Properties of Trig Functions

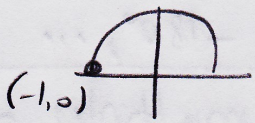


Unit circle
$r = 1$

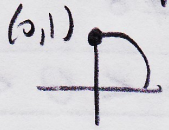
Let t = arclength around unit circle
(Rephrased: t = angle in radians)

Let P = terminal point

Ex: $t = \pi$ corresponds to $P = (-1, 0)$



$P = (0, 1)$ corresponds to $t = \pi/2$



Ex: Find values of $\sin t$, $\cos t$ etc. if $P = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ is on the unit circle

$x = \frac{1}{2}$ $y = -\frac{\sqrt{3}}{2}$ $r = 1$ (on unit circle)

$\sin t = \frac{y}{r} = -\frac{\sqrt{3}}{2}$ $\cos t = \frac{x}{r} = \frac{1}{2}$ $\tan t = \frac{y}{x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\sqrt{3}$

$\csc t = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ $\sec t = 2$ $\cot t = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Function	Domain
$\tan\theta = \frac{y}{x}$, $\sec\theta = \frac{r}{x}$	$x \neq 0$ $\theta \neq 90°, 270°, \ldots$ $-90°, -270°, \ldots$ $\theta \neq$ odd multiples of $90°$
$\csc\theta = \frac{r}{y}$, $\cot\theta = \frac{x}{y}$	$y \neq 0$ $\theta \neq 0°, 180°, 360°, \ldots$ $-180°, \ldots$ $\theta \neq$ multiples of $180°$
$\sin\theta = y/r$, $\cos\theta = x/r$	$r \neq 0$ $-\infty < \theta < \infty$ (r = radius of circle is always > 0)

Periodic Properties

Let k be an integer

$\sin(\theta + 2\pi k) = \sin\theta$

Ex: $\sin\frac{\pi}{2} = \sin\frac{5\pi}{2} = \sin\frac{9\pi}{2} = \sin\frac{-3\pi}{2}$

$+2\pi$

$+4\pi$

-2π

$\tan(\theta + \pi k) = \tan\theta$

Ex: $\tan\frac{\pi}{4} = \tan\frac{5\pi}{4} = \tan\frac{-3\pi}{4} = \tan\frac{-7\pi}{4}$

$+\pi$

$-\pi$

-2π

Periodic Properties

Let k be an integer

$\sin(\theta + 2\pi k) = \sin\theta$
$\cos(\theta + 2\pi k) = \cos\theta$
$\csc(\theta + 2\pi k) = \csc\theta$
$\sec(\theta + 2\pi k) = \sec\theta$

"Period = 2π"

$\tan(\theta + \pi k) = \tan\theta$
$\cot(\theta + \pi k) = \cot\theta$

"Period = π"

Odd-even Properties

$\sin(-\theta) = -\sin\theta$
$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$
$\cot(-\theta) = -\cot\theta$
"odd functions"

$\cos(-\theta) = \cos\theta$
$\sec(-\theta) = \sec\theta$
"even functions"

Ex: Simplify

a) $\sin\left(\frac{-\pi}{6}\right) = -\sin\frac{\pi}{6} = \frac{-1}{2}$ odd function

b) $\cos\left(\frac{-\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ even function

c) $\tan(x+3\pi) = \tan x$ Period $= \pi$

d) $\sin(x+3\pi) = \sin(x+\pi)$ Period $= 2\pi$

e) $\tan 210° = \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ Period $= 180°$

f) $\sin 450° = \sin 90° = \frac{y}{r} = 1$ Period $= 360°$