

6.6 Logarithmic and Exponential Equations P.1

Ex: Solve $\log_3 (2-4x) = 2$

$$3^2 = 2-4x$$

$$7 = -4x$$

$$x = -7/4$$

Domain of $\log_3 (2-4x)$: $2-4x > 0$

Solution is in domain ✓

If equation involves logarithm(s), solutions must be in the domain.

Domain of $\log_b x$ is $x > 0$

Ex: Solve $2\log_2 x = -\log_2 16$

$$\log_2 x = -\frac{1}{2} \log_2 16$$

$$\log_2 x = \log_2 16^{-1/2}$$

Bases are equal $\Rightarrow x = \frac{1}{4}$

Domain: $x > 0$ —

Ex: Solve $\log_7 (x+2) + \log_7 (x+8) = 1$ p.2

$$\log_7 [(x+2)(x+8)] = 1$$

$$7^1 = (x+2)(x+8)$$

$$7 = x^2 + 10x + 16$$

$$x^2 + 10x + 9 = 0$$

$$(x+1)(x+9) = 0$$

$$x = -1, -9$$

Domain: $x+2 > 0$ and $x+8 > 0$

Rules out $x = -9$

$$\boxed{x = -1}$$

Ex: Solve $\ln(x+3) + \ln(x-5) = \ln(x+25)$

$$\ln[(x+3)(x-5)] = \ln(x+25)$$

Bases are equal $\Rightarrow (x+3)(x-5) = x+25$

$$x^2 - 2x - 15 = x + 25$$

$$x^2 - 3x - 40 = 0$$

$$(x-8)(x+5) = 0$$

$$x = 8, -5$$

Domain: $x+3 > 0$ and $x-5 > 0$ and $x+25 > 0$

Rules out $x = -5$

$$\boxed{x = 8}$$

Ex: Solve $4 \cdot 3^x = 8$

p.3

Isolate the
exponential

$$3^x = 2$$

$$\ln 3^x = \ln 2$$

$$x \ln 3 = \ln 2$$

$$x = \frac{\ln 2}{\ln 3} \quad \text{or} \quad \log_3 2$$

Ex: Solve $1.1 = 3.6 e^{t+2}$

$$\frac{1.1}{3.6} = e^{t+2}$$

$$\ln \left(\frac{1.1}{3.6} \right) = \ln e^{t+2}$$

$$\ln \left(\frac{1.1}{3.6} \right) = (t+2) \ln e$$

$$\ln \left(\frac{1.1}{3.6} \right) = t+2$$

$$t = \ln \left(\frac{1.1}{3.6} \right) - 2$$

Ex: Solve $2^{x+1} = 3^x$

$$\ln 2^{x+1} = \ln 3^x$$

$$(x+1) \ln 2 = x \ln 3$$

$$x \ln 2 + \ln 2 = x \ln 3$$

$$\ln 2 = x \ln 3 - x \ln 2$$

$$\ln 2 = x [\ln 3 - \ln 2]$$

$$x = \frac{\ln 2}{\ln 3 - \ln 2}$$

Ex: Solve $9^x - 2 \cdot 3^x - 24 = 0$

Let $3^x = y$

$$9^x = (3^2)^x = 3^{2x} = (3^x)^2 = y^2$$

$$y^2 - 2y - 24 = 0$$

$$(y-6)(y+4) = 0$$

$$\downarrow \quad \downarrow$$

$$y = 6$$

$$\text{or } y = -4$$

$$3^x = 6$$

$$\ln 3^x = \ln 6$$

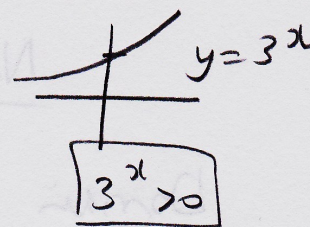
$$x \ln 3 = \ln 6$$

$$x = \frac{\ln 6}{\ln 3}$$

$$x = \frac{\ln 6}{\ln 3}$$

$$3^x = -4$$

no solution



Techniques:

- 1) log form $\rightarrow$ exponential form
- 2) equal bases
- 3) log rules

$$\log_a M + \log_a N = \log_a (MN)$$

etc.

- 4) take $\ln$ of both sides