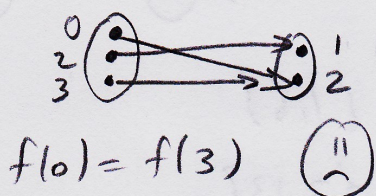


6.2 One-to-One Functions and Inverse Functions

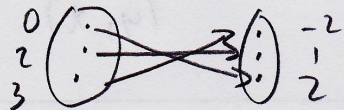
p.1

A function is one-to-one if different x -values never have the same y -value.

Ex: $f = \{(0, 2), (2, 1), (3, 2)\}$ is not one-to-one



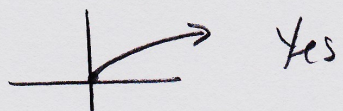
Ex: $g = \{(0, 2), (2, 1), (3, -2)\}$ is one-to-one



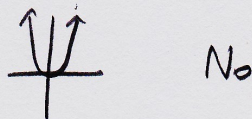
Horizontal Line Test: If every horizontal line intersects the graph in at most one point then the function is one-to-one.

Ex: Is the function one-to-one?

a) $f(x) = \sqrt{x}$



b) $f(x) = x^2$

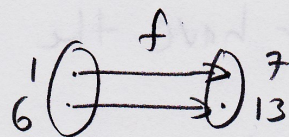


f^{-1} : the inverse of f

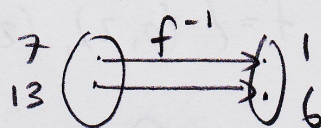
Only defined if f is one-to-one

Intuitively: f^{-1} undoes f

Ex: $f = \{(1, 7), (6, 13)\}$



$f^{-1} = \{(7, 1), (13, 6)\}$



Domain of f = Range of f^{-1} = $\{1, 6\}$

Range of f = Domain of f^{-1} = $\{7, 13\}$

Formally: If f is the set of points (x, y)
 Then f^{-1} " " (y, x)

To verify that Two functions f and g
 are inverses (undo each other) if:

$f(g(x)) = x$ for all x in domain of g

and $g(f(x)) = x$ " " " f

Ex: Verify that the inverse of p.3
 $f(x) = \frac{1}{x+2}$ is $f^{-1}(x) = \frac{1}{x} - 2$

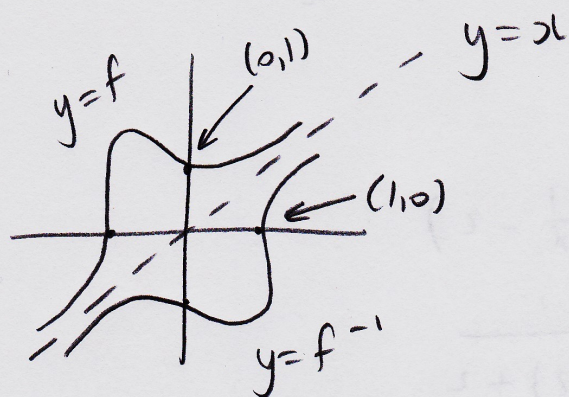
1) $f(f^{-1}(x)) = x$?

$$\begin{aligned} f(f^{-1}(x)) &= f\left(\frac{1}{x} - 2\right) \\ &= \frac{1}{\left(\frac{1}{x} - 2\right) + 2} \\ &= \frac{1}{\left(\frac{1}{x}\right)} \\ &= x \quad \checkmark \end{aligned}$$

2) $f^{-1}(f(x)) = x$?

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}\left(\frac{1}{x+2}\right) \\ &= \frac{1}{\left(\frac{1}{x+2}\right)} - 2 \\ &= x+2 - 2 \\ &= x \quad \checkmark \end{aligned}$$

The graphs of f and f^{-1} are symmetric with respect to $y=x$



Why? x and y -values are swapped

To find the formula for f^{-1} , swap x and y in the formula for f .

Ex: $f(x) = x^2 \quad (x \geq 0)$

Find $f^{-1}(x)$

$$y = x^2 \quad (x \geq 0) \quad (f)$$

$$x = y^2 \quad (y \geq 0) \quad (f^{-1})$$

Swap x
and y

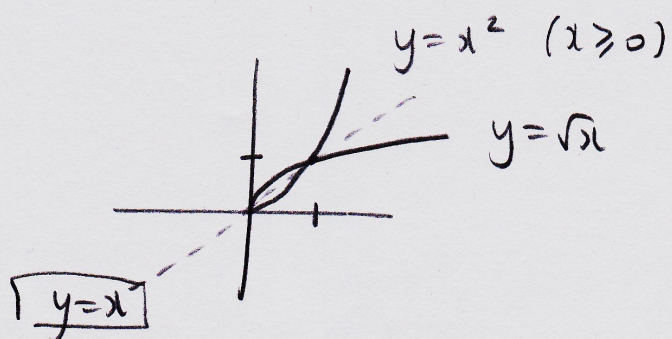
Solve for
 y

$$y^2 = x$$

$$y = \pm \sqrt{x}$$

$$y = \sqrt{x} \text{ because } y \geq 0$$

$$f^{-1}(x) = \sqrt{x}$$



Ex: Find $f^{-1}(x)$ for $f(x) = \frac{3x+1}{x-1}$

$$y = \frac{3x+1}{x-1} \quad (+)$$

Swap
x and y

$$x = \frac{3y+1}{y-1} \quad (f^{-1})$$

Solve for y

$$x(y-1) = 3y+1$$

$$xy - x = 3y + 1$$

$$xy - 3y = x + 1$$

$$(x-3)y = x+1$$

$$y = \frac{x+1}{x-3}$$

$$f^{-1}(x) = \frac{x+1}{x-3} \quad (x \neq 3)$$