

5.4 Polynomial and Rational Inequalities p.1

Ex: Solve $x^5 \leq 6x^3$

1) $y \leq 0$ or $y \geq 0$

$$\underbrace{x^5 - 6x^3}_y \leq 0$$

2) Set $y = 0$ (x-intercepts)

$$y = x^5 - 6x^3$$

$$0 = x^5 - 6x^3$$

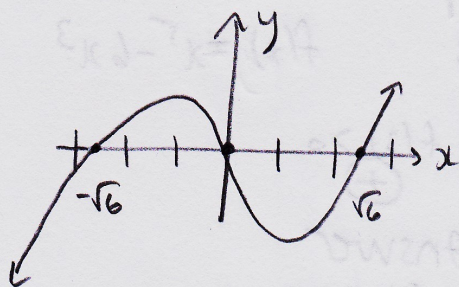
$$0 = x^3(x^2 - 6)$$

$$0 = x^3(x - \sqrt{6})(x + \sqrt{6})$$

$$x = 0, \pm\sqrt{6}$$

$$(0, 0), (\pm\sqrt{6}, 0)$$

3) Graph y



$$\sqrt{6} \approx 2.4$$

End behaviour: $y \approx x^5$



All roots have odd multiplicity
(graph crosses x-axis)

4) Answer

$$y \leq 0 \Rightarrow -\infty < x \leq -\sqrt{6} \text{ or } 0 \leq x \leq \sqrt{6}$$

$$(-\infty, -\sqrt{6}] \cup [0, \sqrt{6}]$$

More Powerful Method:

p.2

Ex: Solve $x^5 \leq 6x^3$

1) $y \leq 0$ or $y \geq 0$

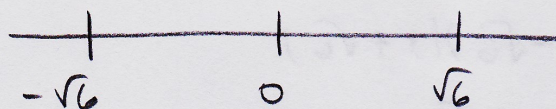
$$\underbrace{x^5 - 6x^3}_{y} \leq 0$$

2) Set $y=0$

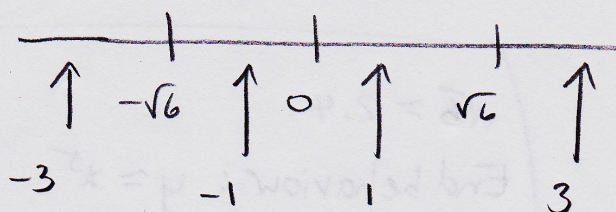
$$x = 0, \pm\sqrt{6}$$

These are the only places that y can change sign.

3) Number line



Test any point in each interval



$$\sqrt{6} \approx 2.4$$

$$f(x) = x^5 - 6x^3$$

$$f(-3) < 0 \ominus \quad f(-1) > 0 \oplus$$

$$f(1) < 0 \ominus$$

$$f(3) > 0 \oplus$$

Means $f(x) < 0$
on the entire interval
 $-\infty < x < -3$

4) Answer
 $f(x) \leq 0$

$$-\infty < x \leq -\sqrt{6} \text{ or } 0 \leq x \leq \sqrt{6}$$

$$(-\infty, -\sqrt{6}] \cup [0, \sqrt{6}]$$

Ex: Solve $x^3 - x^2 - 28x > 2x$ p. 3

1) $y > 0$ or $y < 0$

$$\underbrace{x^3 - x^2 - 30x}_{y} > 0$$

2) Set $y = 0$

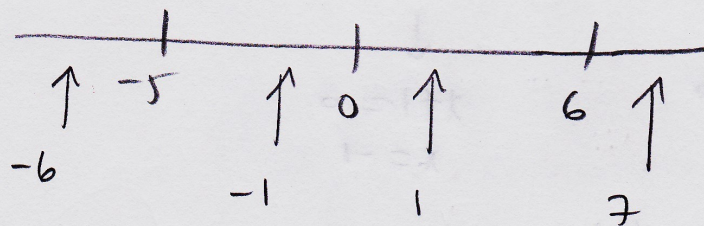
$$0 = x^3 - x^2 - 30x$$

$$0 = x(x^2 - x - 30)$$

$$0 = x(x - 6)(x + 5)$$

$$x = 0, 6, -5$$

3) Number line



$$f(x) = x(x - 6)(x + 5)$$

$$f(-6) = -6(-12)(-1)$$

$$< 0$$

$$\ominus$$

$$f(-1) > 0$$

$$\oplus$$

$$f(1) < 0$$

$$\ominus$$

$$f(7) > 0$$

$$\oplus$$

4) Answer

$$y > 0$$

$$\boxed{(-5, 0) \cup (6, \infty)}$$

A rational function $f(x)$ can only change sign when $f(x)=0$ or undefined. p.4

Caution: Domain of $f(x)$

Ex: Solve $\frac{4x-2}{x+1} \leq 1$

1) $y \leq 0$ or $y \geq 0$

$$\frac{4x-2}{x+1} - \frac{x+1}{x+1} \leq 0$$

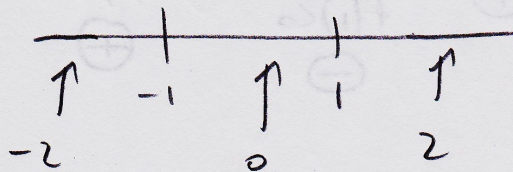
$y \rightarrow \frac{3x-3}{x+1} \leq 0$

2) $y=0$ or undefined?

$\swarrow$
 $3x-3=0$
 $x=1$

$\downarrow$
 $x+1=0$
 $x=-1$

3) Number line



$$f(x) = \frac{3x-3}{x+1}$$

$f(-2) = \frac{-9}{-1} > 0$
 $\oplus$

$f(0) < 0$
 $\ominus$

$f(2) > 0$
 $\oplus$

4) Answer
 $y \leq 0$

p.5

Caution: Domain of $y = x \neq -1$

Include $x=1$ Don't include $x=-1$
because $\leq$

$$-1 < x \leq 1$$

$$(-1, 1]$$