5.3 Graphing Rational Functions

Ex: $f(x) = \dfrac{2x^2 - 2x}{x^2 - x - 2}$

a) Find all intercepts

y-intercept $f(0) = 0$ $(0,0)$

x-intercepts set $f(x) = 0$

$$\frac{2x^2 - 2x}{x^2 - x - 2} = 0$$

$$\frac{2x(x-1)}{(x-2)(x+1)} = 0$$

$$2x(x-1) = 0$$

$$x = 0, 1$$

$(0,0)$ and $(1,0)$

b) Reduce to lowest terms: Domain and V.A.

$$f(x) = \frac{2x(x-1)}{(x-2)(x+1)}$$

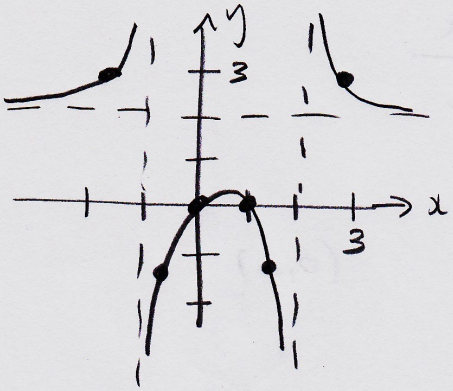
Domain: $x \neq 2, -1$

V.A. $x = 2$, $x = -1$

c) H.A. or O.A.

As $x \to \pm\infty$ $y \to \dfrac{2x^2}{x^2} = 2$ H.A. $y = 2$

d) Plot more points and graph $f(x)$



x	$y = \frac{2x(x-1)}{(x-2)(x+1)}$
-2	$\frac{-4(-3)}{-4(-1)} = 3$
-0.5	$\frac{-1(-1.5)}{-2.5(0.5)} = -1.2$
1.5	$\frac{3(0.5)}{(-0.5)(2.5)} = -1.2$
3	$\frac{6(2)}{1(4)} = 3$

Ex: Caution about domain and V.A.

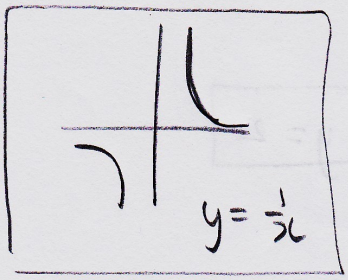
$$y = \frac{x+1}{x^2+x}$$

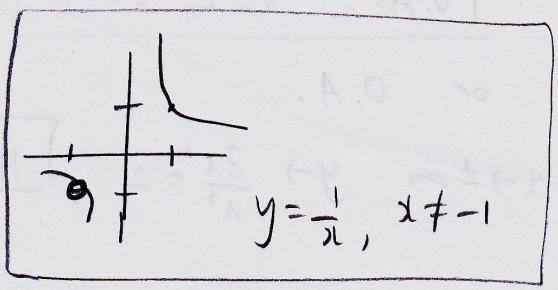
$$= \frac{x+1}{x(x+1)}$$

$$= \frac{1}{x} \quad (x \neq -1)$$

V.A. $x = 0$

Domain: $x \neq 0, -1$





Ex: $y = \dfrac{x^2-4}{x+4}$

a) Find all intercepts

y-intercept $f(0) = -1$ $\quad (0,-1)$

x-intercepts set $f(x)=0$

$$\frac{x^2-4}{x+4} = 0$$

$$\frac{(x-2)(x+2)}{x+4} = 0$$

$$(x-2)(x+2) = 0$$

$$x = \pm 2$$

$(2,0)$, $(-2,0)$ and $(0,-1)$

b) Reduce to lowest terms: Domain and V.A.

$$f(x) = \frac{(x-2)(x+2)}{x+4}$$

V.A. $x=-4$

Domain: $x \neq -4$

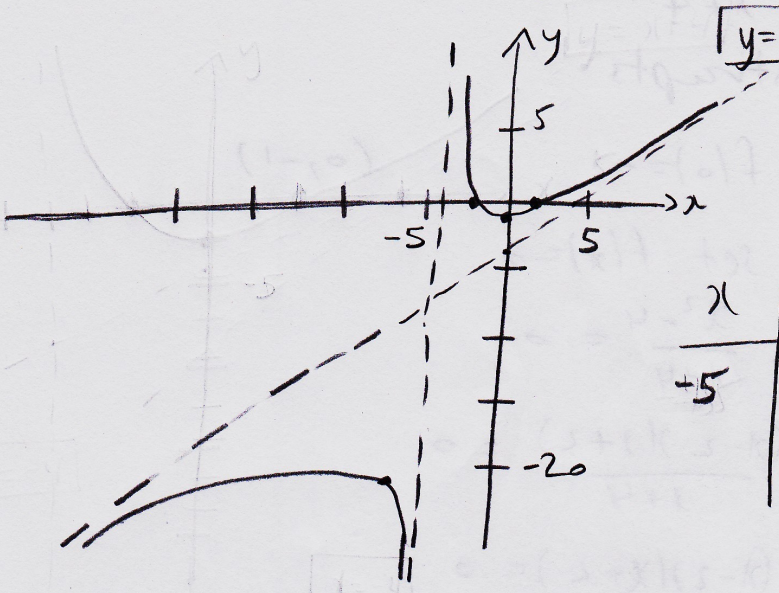
c) H.A. or O.A.

$$\begin{array}{r} x-4 \\ x+4 \overline{)\,x^2+0x-4} \\ -(x^2+4x) \\ \hline -4x-4 \\ -(-4x-16) \\ \hline 12 \end{array}$$

← Quotient

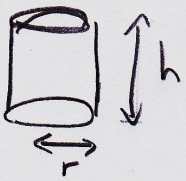
O.A. $y = x-4$

d) More points and graph $f(x)$



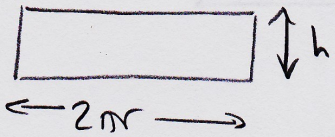
x	$y = \frac{x^2-4}{x+4}$
$+5$	$21/-1 = -21$

Ex: Volume of a cylinder is $100\,cm^3$. Find the cylinder's surface area as a function of its radius.



$$SA = 2(\pi r^2) + 2\pi rh$$

$2(\pi r^2)$: top and bottom; $2\pi rh$: side



Given $V = 100$

$\pi r^2 h = 100$

$h = \frac{100}{\pi r^2}$

$$SA = 2\pi r^2 + 2\pi r \cdot \frac{100}{\pi r^2}$$

$$= 2\pi r^2 + \frac{200}{r}$$

$$= \frac{2\pi r^3 + 200}{r} \quad \left(\cdot \frac{r}{r}\right)$$

or $SA = \frac{2(\pi r^3 + 100)}{r}$