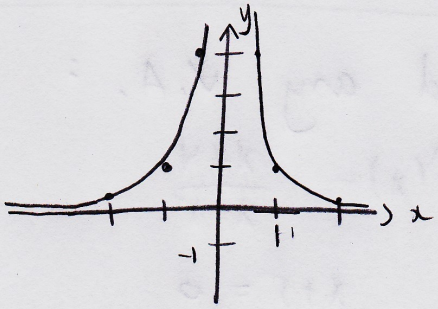
5.2 Properties of Rational Functions

Rational function $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

Ex: Graph $f(x) = \frac{1}{(x-3)^2} + 1$ using transformations

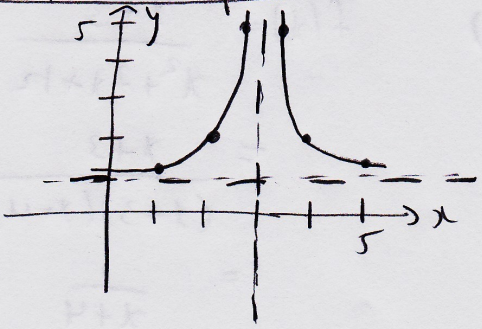
x	$y = \frac{1}{x^2}$
-2	$\frac{1}{4}$
-1	1
$-\frac{1}{2}$	$\frac{1}{(\frac{1}{4})} = 4$
0	undefined
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$



$y = \frac{1}{x^2}$

Shift 3 units right
1 unit up

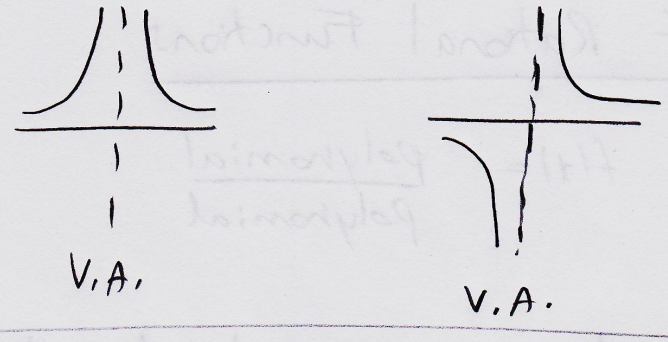
$y = \frac{1}{(x-3)^2} + 1$



The vertical line $x = c$ is a vertical asymptote of $f(x)$ if $f(x) \to \pm\infty$ as $x \to c$

Ex: $\frac{1}{x^2}$ has a V.A. at $x = 0$

$\frac{1}{(x-3)^2} + 1$ " at $x = 3$



To find V.A. : reduce $f(x)$ to lowest terms and find zeros of denominator

Ex: Find any V.A. :

a) $f(x) = \frac{x+4}{x+5}$

$x+5 = 0$

$x = -5$

$x=-5$ is a V.A.

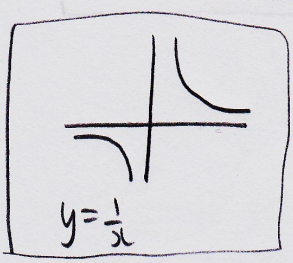
b) $f(x) = \frac{x+3}{x^2+7x+12}$

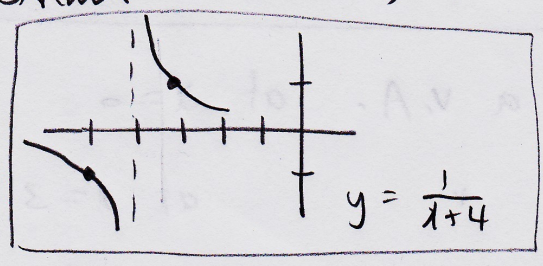
$= \frac{x+3}{(x+3)(x+4)}$

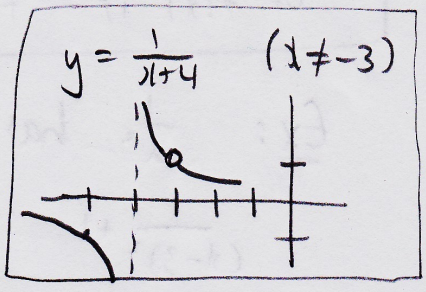
$= \frac{1}{x+4} \quad (x \neq -3)$

V.A. $x=-4$

Domain : $x \neq -3, -4$



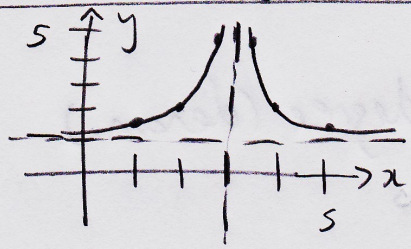




Horizontal Asymptote $y = \#$

As $x \to \pm\infty$, $y \to \#$

Ex:

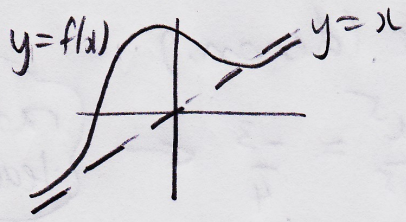


$y = \frac{1}{(x-3)^2} + 1$ has a H.A. at $y = 1$

Oblique Asymptote $y = mx + b$

As $x \to \pm\infty$, $y \to mx + b$

Ex:



$f(x)$ has an O.A. $y = x$

If degree (num.) < degree (denom.) $f(x)$ has H.A. $y = 0$

degree (num.) = degree (denom.) " H.A. (take ratio)

degree (num.) = degree (denom.) + 1 " O.A. (long division)

degree (num.) > degree (denom.) + 1 " neither

Ex: Find any H.A. or O.A.

a) $y = \dfrac{4x^6+1}{x^9-2}$

degree (num.) < degree (denom.)

As $x \to \pm\infty$ $\quad y \to 0$

H.A. $y=0$

b) $y = \dfrac{-6x^5+4}{8x^5+1}$

degree (num.) = degree (denom.)

As $x \to \pm\infty$ $\quad y \to \dfrac{-6x^5}{8x^5} = \dfrac{-3}{4}$ ← ratio of leading terms

H.A. $y = \frac{-3}{4}$

c) $y = \dfrac{2x^3-5x^2}{x^2+2}$

degree (num.) = degree (denom.) + 1

LONG DIVISION

$$\begin{array}{r} 2x-5 \\ (x^2+2)\overline{)\,2x^3-5x^2+0x+0} \\ -(2x^3 \qquad +4x) \\ \hline -5x^2-4x+0 \\ -(-5x^2 \qquad -10) \\ \hline -4x+10 \end{array}$$

$$\frac{2x^3-5x^2}{x^2+2} = \text{quotient} + \frac{\text{remainder}}{x^2+2}$$

O.A.

$$= 2x-5 + \frac{-4x+10}{x^2+2}$$

As $x \to \pm\infty$, $y \to 2x-5$

O.A. $y = 2x-5$

d) $$y = \frac{x^9-2}{4x^6+1}$$

degree (num.) > degree (denom.) + 1

Neither

e) $$y = \frac{4x^2-7}{2x+6}$$

degree (num.) = degree (denom.) + 1

LONG DIVISION

$$\begin{array}{r} 2x-6 \\ (2x+6)\overline{)\,4x^2+0x-7} \\ -(4x^2+12x) \\ \hline -12x-7 \\ -(-12x-36) \\ \hline 29 \end{array}$$

Quotient = $2x-6$

O.A. $y = 2x-6$