5.1 Polynomial Function

$f(x) = a_n x^n + \dots + a_1 x + a_0$

$a_n, \dots, a_0$: coefficients: any real #

powers of x: non-negative integers

degree: highest power of x

Ex: Polynomials:

x^4 has degree 4

$x^3 - \sqrt{3}x^2$ 3

$2x$ 1

7 0

0 has no degree

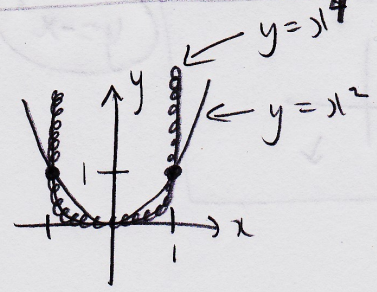
$(x^2+1)(x-3)^2$ has degree 4

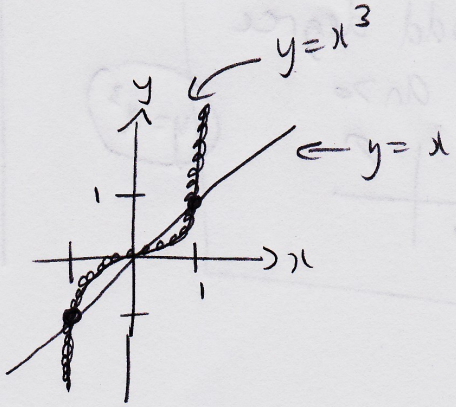
Not polynomials:

$\sqrt{x}$

$\frac{x+1}{x-2}$

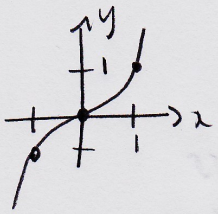
Ex:



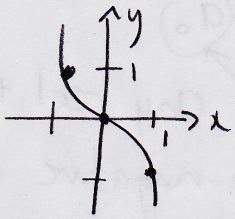


As exponent increases, graph is flatter near origin and steeper elsewhere.

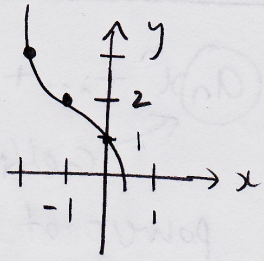
Ex: Graph $f(x) = -(x+1)^3 + 2$ using transformations



$y = x^3$



$y = -x^3$



$y = -(x+1)^3 + 2$

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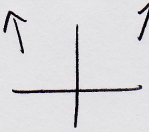
4 Possible end behaviours for a polynomial

$y = a_n x^n + \ldots + a_0$

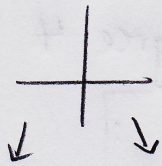
even degree
$a_n > 0$



$f(x) \to \infty$ as $x \to \pm\infty$

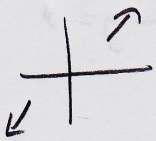
$y = x^2$

even degree
$a_n < 0$



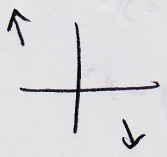
$y = -x^2$

odd degree
$a_n > 0$



$y = x^3$

odd degree
$a_n < 0$



$y = -x^3$

If r is a real number for which $f(r) = 0$ then r is called a <u>real zero</u> of $f(x)$.

The following are equivalent:

- r is a real zero of $f(x)$
- $f(r) = 0$
- r is an x-intercept of $f(x)$
- $x - r$ is a factor of $f(x)$

<u>Ex</u>: a) Find a polynomial of degree 3 whose zeros are $-7, 0$ and 3

$f(x) = a(x+7)x(x-3)$

↖ any real # $\neq 0$

b) Find the coefficient if $f(x)$ passes through $(1, -32)$

Sub $y = -32$, $x = 1$: $-32 = a(8)(1)(-2)$

$a = 2$

$f(x) = 2(x+7)x(x-3)$

Ex: $f(x) = 12(x-4)^2(x+5)$

x-intercepts: Set $y=0$

$0 = 12(x-4)^2(x+5)$

$x = 4, -5$

$x=4$ is a real zero of multiplicity 2

$x=-5$ " 1

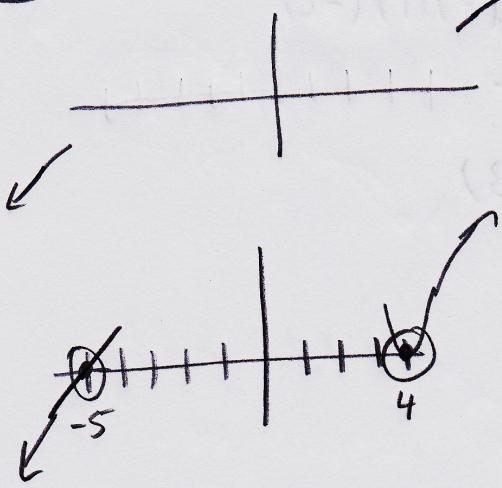
Multiplicity of a zero $x=r$: exponent on the factor $x-r$

At a real zero of **odd** multiplicity, $f(x)$ **crosses** the x-axis

" **even** multiplicity, $f(x)$ **touches** "

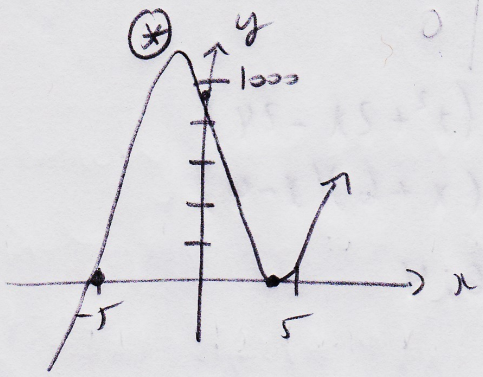
Ex: $f(x) = 12(x-4)^2(x+5)$

$f(x) \approx 12x^3$ (end behavior)



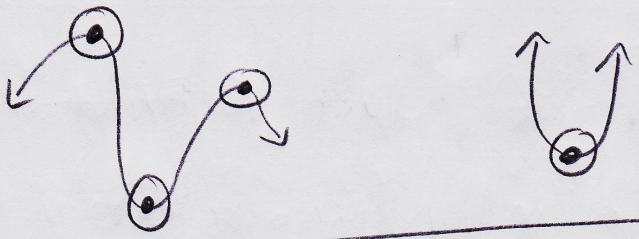
More points

x	y
0	$12(-4)^2(5) = 960$



⊛ exact point unknown

Turning Points: Local Max or Local Min



turning points ≤ degree - 1

Ex: a) $f(x)$ has degree 7

$\Rightarrow$ # turning points ≤ 6

b) $f(x)$ has 2 turning points

$\Rightarrow$ degree ≥ 3

Ex: $f(x) = x^4 + 2x^3 - 24x^2$

a) Find x-intercepts

Set $y=0$: $0 = x^2(x^2+2x-24)$

$0 = x^2(x+6)(x-4)$

$x = 0, -6, 4$

b) Multiplicities

$x=0$ has multiplicity	2
$x=-6$	1
$x=4$	1

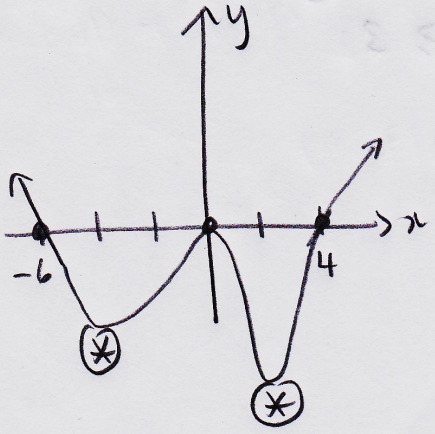
c) End behaviour

$f(x) \approx x^4$

d) # turning points $\leq$ degree -1

≤ 3

e) Graph $f(x)$



(*) exact points unknown