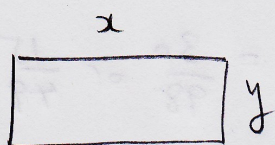


4.4 Quadratic Models

p.1

Ex: Enclose a rectangular area with 3,000m of fence. Find dimensions that maximize the area. What is the maximum area?



$$2x + 2y = 3,000$$

$$2y = 3,000 - 2x$$

$$y = 1500 - x$$

Maximize area $A = xy$

$$\begin{aligned} A(x) &= x(1500 - x) \\ &= -x^2 + 1500x \end{aligned}$$

Parabola ✓
Find vertex

$$x = \frac{-b}{2a} = \frac{-1500}{-2} = 750$$

$$y = 1500 - x = 750$$

$$A = xy = 562,500$$

Maximum area is $562,500 \text{ m}^2$ when dimensions are $750 \text{ m} \times 750 \text{ m}$

Ex: Height (in m) of a ball after
t seconds is $h(t) = -4.9t^2 + 3t + 5$

p.2

a) When does ball reach maximum height?

Parabola —
Find vertex

$$x = \frac{-b}{2a} \quad t = \frac{-b}{2a} = \frac{-3}{-9.8} = \frac{30}{98} \text{ or } \frac{15}{49}$$

$$\boxed{\text{At } \frac{15}{49} \text{ seconds}}$$

b) What is the maximum height?

$$h\left(\frac{15}{49}\right) = -4.9\left(\frac{15}{49}\right)^2 + 3\left(\frac{15}{49}\right) + 5$$
$$\approx \boxed{5.46 \text{ m}}$$

c) When does the ball hit the ground?

↖
 $h = 0$

Set $h = 0$

$$0 = -4.9t^2 + 3t + 5$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

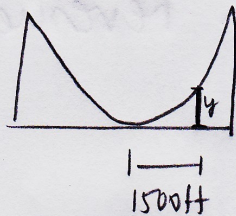
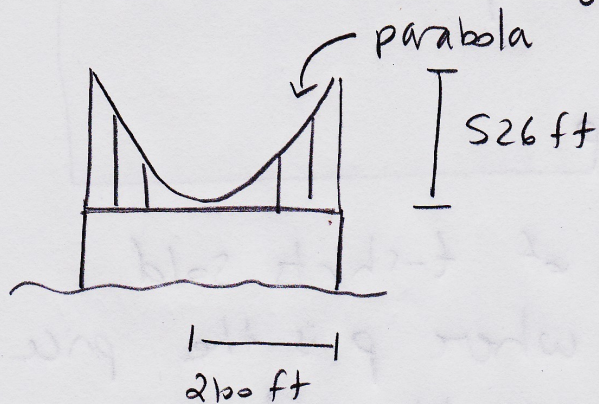
$$= \frac{-3 \pm \sqrt{107}}{-9.8}$$

$$\approx 1.36, -0.75$$

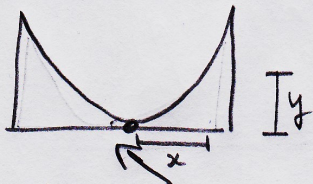
$$\boxed{\text{At } 1.36 \text{ seconds}}$$

Ex: Golden Gate Bridge

p. 3



Find y .



Let this be $(0,0)$

Standard form for parabola: $y = a(x-h)^2 + k$

Vertex $= (h, k) = (0, 0)$

$$y = ax^2$$

$$\text{Sub } y = 526 ;$$
$$x = 2100$$

$$526 = a(2100)^2$$

$$a = \frac{526}{2100^2}$$

$$y = \frac{526}{2100^2} x^2$$

$$\text{Sub } x = 1500 : y = \frac{526}{2100^2} \cdot 1500^2 \approx 268.37 \text{ ft}$$

Revenue = (Quantity Sold) · (Price)

\$ received

$$R = xp$$

p. 4

Ex: The quantity of t-shirts sold is $x = -20p + 600$, where p is the price in dollars. What quantity and price maximize revenue? Maximum revenue?

$$\begin{aligned} R &= xp \\ R(p) &= (-20p + 600)p \\ &= -20p^2 + 600p \end{aligned}$$

Parabola ✓
Find vertex

$$p = \frac{-b}{2a} = \frac{-600}{-40} = 15$$

$$x = -20p + 600 = -20(15) + 600 = 300$$

$$R(15) = xp = 300(15) = 4500$$

Max. revenue is \$4500, when
300 t-shirts are sold at a price of \$15.