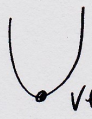
4.3 Quadratic Functions

$f(x) = ax^2 + bx + c$ (parabolas)

$a > 0$



Parabola opens up

Vertex is lowest point

Function has a minimum y-value

$a < 0$

Opens down

Vertex is highest point

Function has a maximum y-value

Axis of symmetry is $x = \frac{-b}{2a}$

Ex: Find axis of symmetry and vertex

$y = 2x^2 + 6x + 10$

axis $x = \frac{-b}{2a} = \frac{-6}{4} = \frac{-3}{2}$

$\boxed{x = \frac{-3}{2}}$

Vertex $= \left(-\frac{3}{2}, f\left(\frac{-3}{2}\right)\right)$

$f\left(-\frac{3}{2}\right) = 2\left(\frac{-3}{2}\right)^2 + 6\left(\frac{-3}{2}\right) + 10$

$= 2\left(\frac{9}{4}\right) - \frac{18}{2} + \frac{20}{2}$

$= \frac{9}{2} + \frac{2}{2}$

$= \frac{11}{2}$

$\boxed{\text{Vertex} = \left(-\frac{3}{2}, \frac{11}{2}\right)}$

Ex: Consider $y = 2x^2 + 6x + 10$

a) Opens up or down?

$a > 0 \Rightarrow$ opens up

b) Find all intercepts

y-intercept: set $x = 0$

$y = 10$

$\boxed{(0, 10)}$

x-intercepts: set $y = 0$

$0 = 2x^2 + 6x + 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

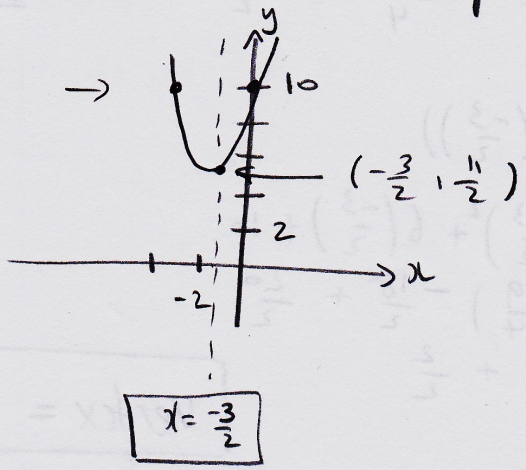
$$= \frac{-6 \pm \sqrt{36 - 80}}{4} \leftarrow \text{no real solution}$$

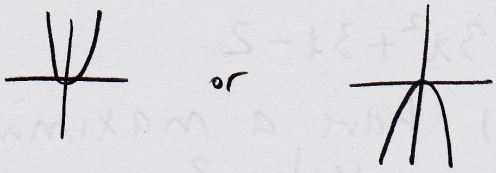
$\boxed{\text{No } x\text{-intercepts}}$ (graph does not cross x-axis)

Ex: Graph $y = 2x^2 + 6x + 10$

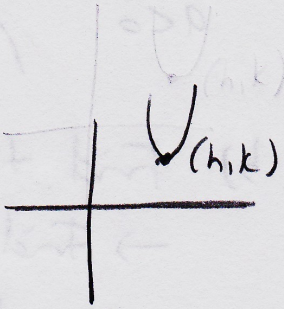
1) Opens up 2) Axis $x = -\frac{3}{2}$

3) Vertex $= (-\frac{3}{2}, \frac{11}{2})$ 4) Intercepts: $(0, 10)$



$y = ax^2$:  or

Shift right h units
and up k units



$$y = a(x-h)^2 + k$$
standard form for parabola

Ex: Parabola has vertex $(-2,6)$ and passes through $(1,51)$. Find $y = ax^2+bx+c$

Standard form $y = a(x-h)^2+k$

$(h,k) = (-2,6)$

$$y = a(x+2)^2+6$$

Sub $x=1$, $y=51$ to get a :

$51 = a(3)^2 + 6$

$45 = 9a$

$a = 5$

$$y = 5(x+2)^2+6$$

$y = 5(x^2+4x+4)+6$

$y = 5x^2+20x+26$

Ex: $f(x) = -3x^2 + 3x - 2$

a) Does $f(x)$ have a maximum or minimum value?

$a < 0$ ∩ **Maximum**

b) Find the extreme value of $f(x)$

→ Find the vertex

$$x = \frac{-b}{2a} = \frac{-3}{-6} = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = -3\left(\frac{1}{4}\right) + 3\left(\frac{1}{2}\right) - 2$$

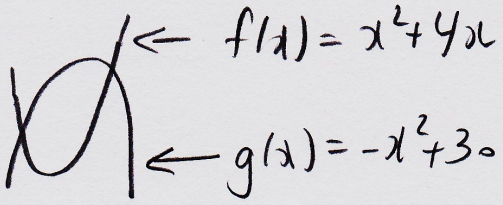
$$= -\frac{3}{4} + \frac{6}{4} - \frac{8}{4}$$

$$= -\frac{5}{4}$$

Vertex $= \left(\frac{1}{2}, \frac{-5}{4}\right)$ ← y

Maximum value of $-\frac{5}{4}$ at $x = \frac{1}{2}$.

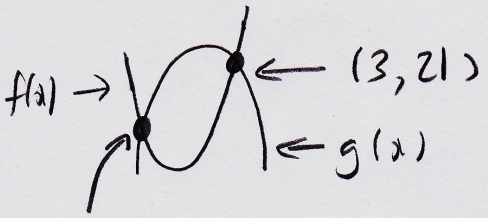
Ex:



a) Solve $f(x) = g(x)$

Want x-value(s)

$$x^2 + 4x = -x^2 + 30$$
$$2x^2 + 4x - 30 = 0$$
$$2(x^2 + 2x - 15) = 0$$
$$2(x+5)(x-3) = 0$$
$$x = -5, 3$$



(-5, 5)

b) Solve $f(x) < g(x)$

Want x-value(s)

$$-5 < x < 3$$