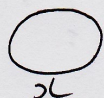


### 3.6 Building Functions

p.1

Ex: A wire of length  $x$  is bent into a circle. Express the circle's area as a function of  $x$ .



$$C = x$$

$$2\pi r = x$$

$$r = \frac{x}{2\pi}$$

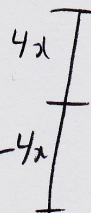
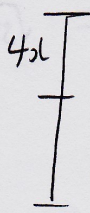
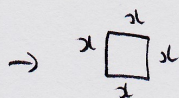
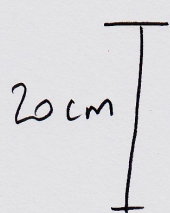
$$A = \pi r^2$$

$$= \pi \left( \frac{x}{2\pi} \right)^2$$

$$= \pi \frac{x^2}{4\pi^2}$$

$$= \frac{x^2}{4\pi}$$

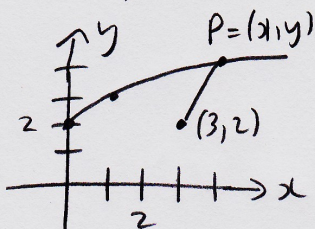
Ex: A wire of length 20 cm is cut into 2 pieces and bent into a square and a circle. Express the circle's circumference as a function of the square's side length,  $x$ .



$$C = 20 - 4x$$

Ex: Let  $P = (x, y)$  be a point on the graph of  $y = \sqrt{x} + 2$ . p.2

- a) Express the distance from  $P$  to the point  $(3, 2)$  as a function of  $x$ .



distance from  $(3, 2)$  to  $(x, y) =$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x - 3)^2 + (y - 2)^2}$$

$$y = \sqrt{x} + 2$$

$$= \sqrt{(x - 3)^2 + (\sqrt{x})^2}$$

$$= \sqrt{x^2 - 6x + 9 + x}$$

$$= \sqrt{x^2 - 5x + 9}$$

- b) Find the distance when  $x = 4$

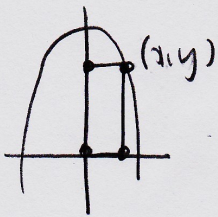
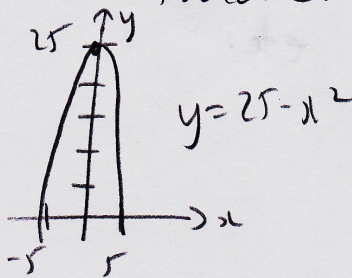
$$d = \sqrt{4^2 - 5(4) + 9}$$

$$= \sqrt{25 - 20}$$

$$= \sqrt{5}$$

Ex: A rectangle is in Quadrant I with one corner: at the origin  
on the positive  $x$ -axis  
"  $y$ -axis

on the graph  $y = 25 - x^2$   
a) Call the upper right corner  $(x, y)$ .  
Express the rectangle's area as a function of  $x$ .



$$\begin{aligned}\text{Area} &= \text{base} \cdot \text{height} \\ &= x \cdot y\end{aligned}$$

$$\begin{aligned}&= x(25 - x^2) \\ &= 25x - x^3\end{aligned}$$

b) Domain of the function?  
( $x$ -values?)

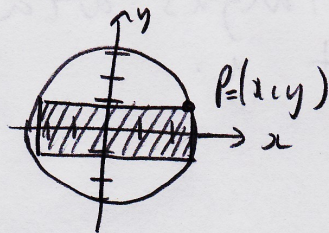
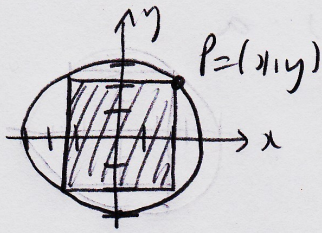
$$x > 0 \quad \text{and}$$

$$\begin{aligned}y &> 0 \\ -5 < x < 5\end{aligned}$$

Together:

$$\boxed{0 < x < 5}$$

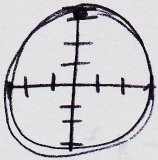
Ex: A rectangle is inscribed in the circle of radius 3 centred at the origin, with sides parallel to the axes. Let  $P=(x,y)$  be the upper right corner. Find the rectangle's area as a function of  $x$ .



etc.

$$\begin{aligned} A &= \text{base} \cdot \text{height} \\ &= (2x)(2y) \\ &= 4xy \end{aligned}$$

$$\leftarrow = 4x \sqrt{9 - x^2}$$



$$x^2 + y^2 = 9$$

$$y^2 = 9 - x^2$$

$$y = \pm \sqrt{9 - x^2}$$

