

3.5 Graphing Transformations

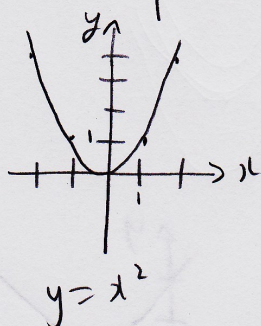
p.1

Vertical Shift : Let $k > 0$

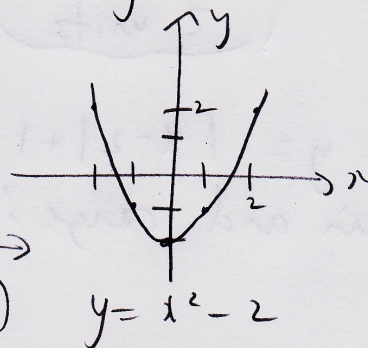
$y = f(x) + k$ is $y = f(x)$ shifted up by k units

$y = f(x) - k$ " " down "

Ex: Graph $y = x^2$ and $y = x^2 - 2$



Shift
down by 2



Horizontal Shift : Let $h > 0$

$y = f(x - h)$ is $y = f(x)$ shifted right by h units

$y = f(x + h)$

"

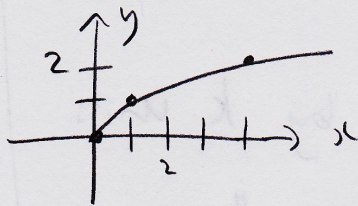
left

"

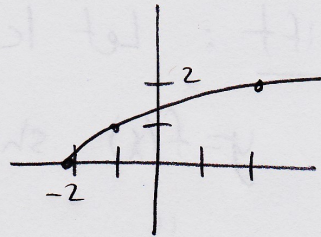
means x is
replaced by $x + h$

Ex: Graph $y = \sqrt{x}$ and $y = \sqrt{x+2}$

p.2



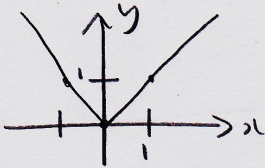
$$y = \sqrt{x}$$



$$y = \sqrt{x+2}$$

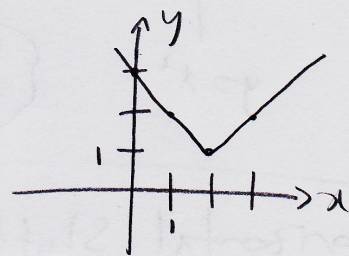
Shift left
2 units

Ex: Graph $y = |x-2|+1$
Domain and range?



$$y = |x|$$

Shift right by 2
and up by 1



$$y = |x-2|+1$$

Domain : $(-\infty, \infty)$
(x-values)

Range : $[1, \infty)$
(y-values)

Reflections :

$y = -f(x)$ is $y = f(x)$ reflected in x -axis

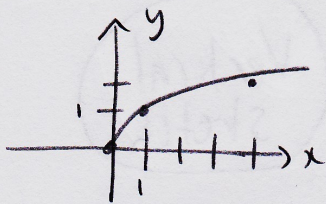
$$y = f(-x)$$

"

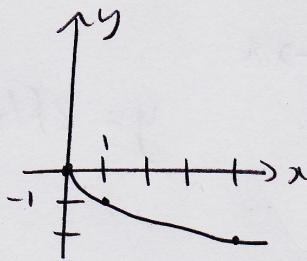
y-axis

means replace x with $-x$

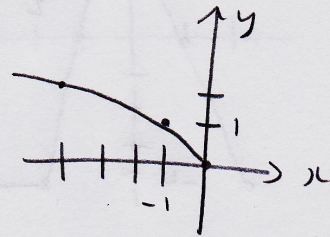
Ex: Graph $y = \sqrt{x}$, $y = -\sqrt{x}$ and $y = \sqrt{-x}$



$$y = \sqrt{x}$$



$$y = -\sqrt{x}$$



$$y = \sqrt{-x}$$

Reflect in
 x -axis

Reflect
in y -axis

Vertical Stretch/Compression: Let $a > 0$
 $y = af(x)$ multiplies each y -coordinate
 of $y = f(x)$ by a

If $a > 1$ it is a vertical stretch

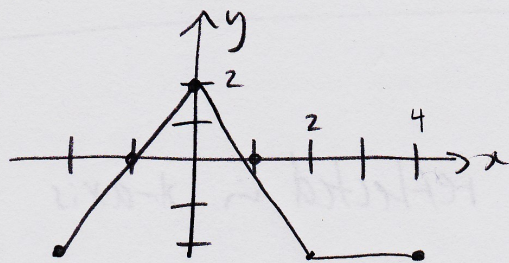
$$0 < a < 1$$

"

compression

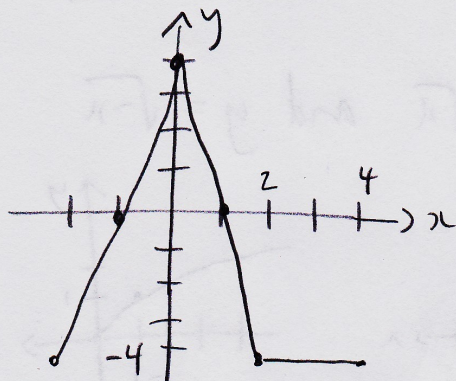
Ex:

p. 4



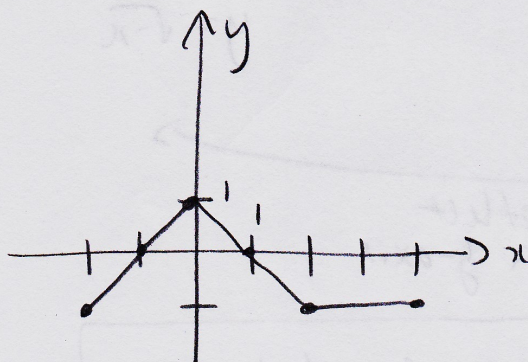
$$y = f(x)$$

Graph $y = 2f(x)$ and $y = \frac{1}{2}f(x)$



$$y = 2f(x)$$

Vertical
stretch



$$y = \frac{1}{2}f(x)$$

Vertical
Compression

Horizontal Stretch/Compression: Let $a > 0$

$y = f(ax)$ divides each x-coordinate of $y = f(x)$ by a

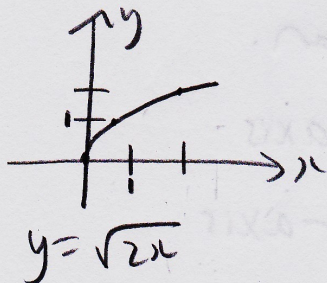
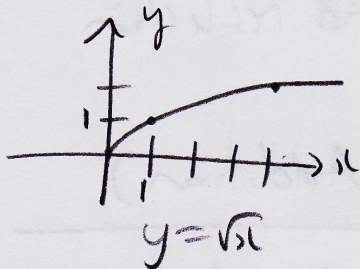
If $a > 1$ it is a horizontal compression

$0 < a < 1$

"

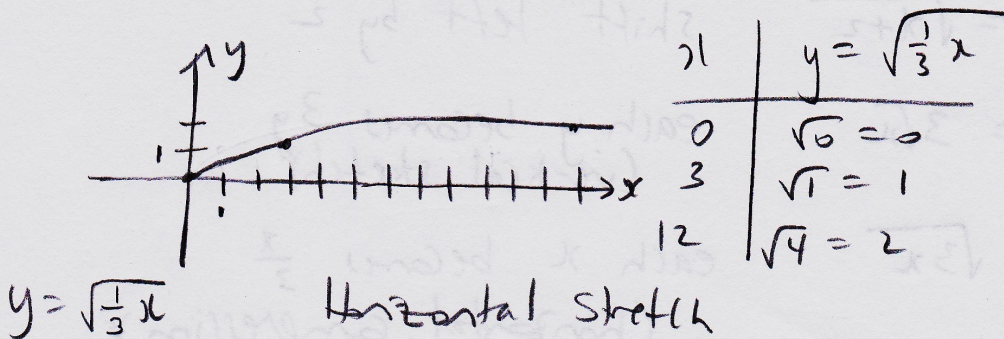
stretch

Ex: Graph $y = \sqrt{x}$, $y = \sqrt{2x}$, $y = \sqrt{\frac{1}{3}x}$



Horizontal Compression

x	$y = \sqrt{2x}$
0	$\sqrt{0} = 0$
$\frac{1}{2}$	$\sqrt{1} = 1$
2	$\sqrt{4} = 2$



Horizontal Stretch

x	$y = \sqrt{\frac{1}{3}x}$
0	$\sqrt{0} = 0$
3	$\sqrt{1} = 1$
12	$\sqrt{4} = 2$

Standard Form

p.6

$$y = a f(b(x \pm c)) \pm d$$

Vertical Shift

Hor. Shift

Vertical Stretch/Compression
Reflect in x-axis if $a < 0$

Horizontal Stretch/Compression
Reflect in y-axis if $b < 0$

Note: Do stretch/compression and reflection before shifting

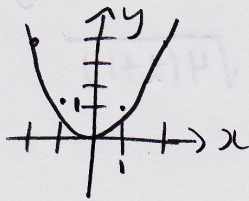
(Multiplication Before Addition)

Ex: Describe in words how to transform $y = \sqrt{x}$ to get the new function.

- a) $y = -\sqrt{x}$ reflect in x-axis
- b) $y = \sqrt{-x}$ reflect in y-axis
- c) $y = \sqrt{x} + 2$ shift up by 2
- d) $y = \sqrt{x+2}$ shift left by 2
- e) $y = 3\sqrt{x}$ each y becomes 3y
(vertical stretch)
- f) $y = \sqrt{3x}$ each x becomes $\frac{x}{3}$
(horizontal compression)

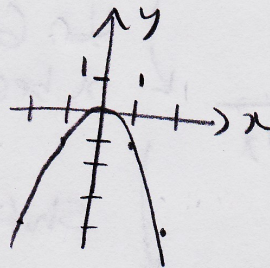
Ex: Graph $y = -x^2 + 2$ using transformations p.7

$$y = x^2$$



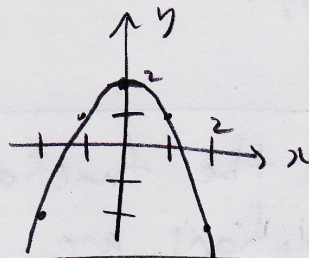
$$y = -x^2$$

Reflect in x-axis



$$y = -x^2 + 2$$

Shift up by 2

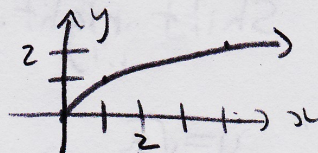


Ex: Graph $y = \sqrt{-(x+3)} + 2$ using transformations

$$y = \sqrt{x}$$

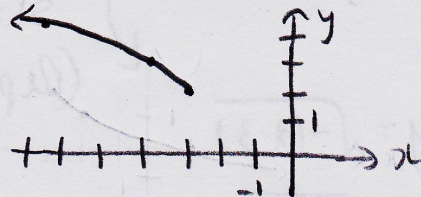
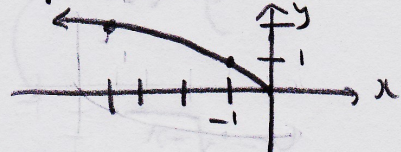
$$y = \sqrt{-x}$$

Reflect in y-axis



$$y = \sqrt{-(x+3)} + 2$$

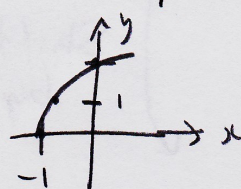
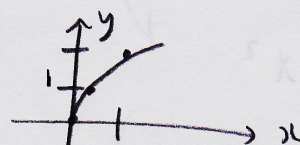
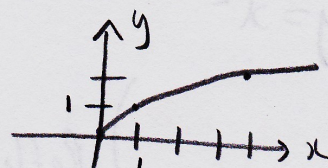
Shift left by 3 and up by 2



Ex: Graph $y = \sqrt{4x+4}$ using transformations p.8

Standard Form $y = \sqrt{4(x+1)}$

$$\begin{aligned} y &= \sqrt{x} \\ &\downarrow \text{Hor. Compression} \\ &\quad x \text{ becomes } \frac{x}{4} \\ y &= \sqrt{4x} \\ &\downarrow \text{Shift Left by 1} \\ y &= \sqrt{4(x+1)} \end{aligned}$$



Ex: Find the function if the following transformations are applied (in order) to $y = \sqrt{x}$

1. Reflect in y-axis
2. Shift right 3 units

$$\begin{aligned} y &= \sqrt{x} \\ &\downarrow \text{Reflect in y-axis} \\ &\quad (\text{Replace } x \text{ by } -x) \\ y &= \sqrt{-x} \\ &\downarrow \text{Shift right by 3} \\ &\quad (\text{Replace } x \text{ by } x-3) \end{aligned}$$

$$\text{or } y = \sqrt{-(x-3)}$$

$$\text{or } y = \sqrt{-x+3}$$

$$\text{or } y = \sqrt{3-x}$$