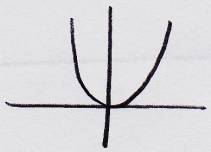
3.3 Properties of Functions

Even function: $f(-x) = f(x)$

Symmetric about y-axis

Ex: $f(x) = x^2$ is even

$f(-x) = (-x)^2 = x^2 = f(x)$



$y = x^2$

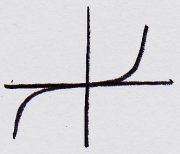
Symmetric about y-axis ✓

Odd function: $f(-x) = -f(x)$

Symmetric about origin

Ex: $f(x) = x^3$ is odd

$f(-x) = (-x)^3 = -x^3 = -f(x)$



$y = x^3$

Symmetric about origin ✓

(Unchanged when rotated 180° about origin)

Ex: Even, odd or neither?

a) $f(x) = x^2 + 2x$

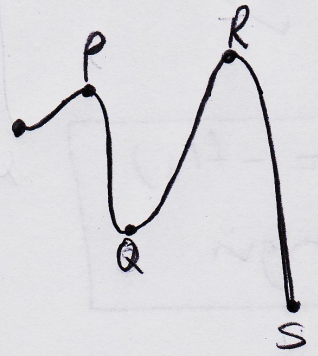
$f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$ neither

b) $f(x) = x^3 - 6x$

$f(-x) = (-x)^3 - 6(-x) = -x^3 + 6x = -(x^3 - 6x)$ odd

c) $f(x) = |x|$

$f(-x) = |-x| = |x| = f(x)$ even



(Hilltops)

Local maximum points: P, R

(Highest)

Absolute maximum point: R

Local minimum points: Q

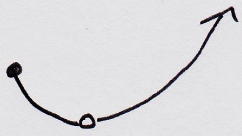
(Valley floors)

Absolute minimum point: S

(Lowest)

Note: An endpoint can be an absolute max/min but not a local max/min.

Ex:

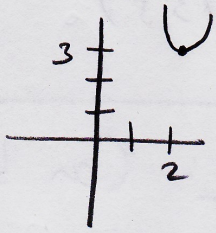


No local max (no hilltop)

No local or absolute min (hole)

No absolute max (f increases to $+\infty$)

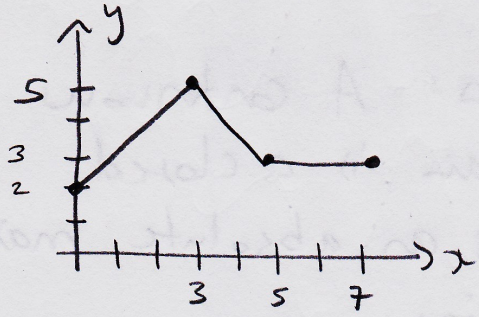
Terminology:



Function has a local minimum of 3 at $x=2$

(3 is the y-value)

Ex:



a) Absolute max?

Absolute max of 5 at $x=3$

b) Absolute min?

Absolute min of 2 at $x=0$

c) where is f increasing? (Where is y increasing?)

$0 < x < 3$

On the interval $(0, 3)$ (Use open intervals)

d) Where is f constant?

$5 < x < 7$

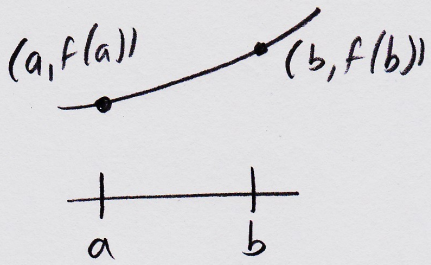
On the interval $(5, 7)$

e) where is f decreasing?

On the interval $(3, 5)$

Continuous function: Can be traced without lifting pencil (no jumps or holes)

Extreme Value Theorem: A continuous function whose domain is a closed interval $[a, b]$ has an absolute max and an absolute min.



Average rate of change of f from a to b

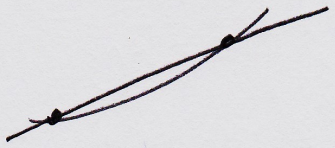
$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{f(b)-f(a)}{b-a}$$

Ex: Find average rate of change of $f(x)=5x^2+2$ from $x=2$ to $x=4$

$$\frac{\Delta y}{\Delta x} = \frac{f(4)-f(2)}{4-2} = \frac{82-22}{2} = 30$$

If y: °C x: minutes
Units would be °C/min



m_{sec} = av. rate of change of f

Ex: The average rate of change from $(1,-2)$ to $(b, f(b))$ is 4. Find equation of secant line from $(1,-2)$ to $(b, f(b))$.

$m_{sec} = 4$ $\quad (x_1, y_1) = (1,-2)$

$y-y_1 = m(x-x_1)$

$y+2 = 4(x-1)$ $\quad \boxed{y = 4x-6}$