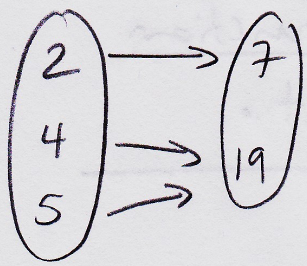
# 3.1 Functions

**Relation**: Mapping between 2 sets

**Ex**: Relation $f$ given by:

$f(2)=7$, $f(4)=19$, $f(5)=19$

As a mapping:




As a set of ordered pairs:

$\{(2,7), (4,19), (5,19)\}$

**Domain**: Set of inputs

**Range**: Set of outputs

A relation is a **function** if each element in the domain is mapped to exactly 1 element in the range.

**Ex**: Relation $f$ above

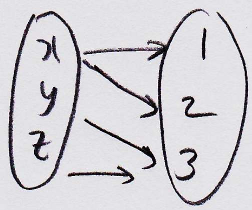
Domain $= \{2,4,5\}$

Range $= \{7,19\}$

$f$ is a function ✓

Ex: A relation that is not a function

As a mapping




Ordered pairs

$\{(x,1),(x,2),$
$(y,3),(z,3)\}$

$x$ is mapped to 2 different elements

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Intuitively: The output of a function is predictable for each input.

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Ex: Is $y$ a function of $x$?

a) $y=3x+7$

$\boxed{\text{Yes}}$ Each input $x$ gets mapped to 1 $y$-value

b) $y-x^2-1=0$

$y=x^2+1$

$\boxed{\text{Yes}}$

c) $x^2+y^2=4$

$y^2=4-x^2$

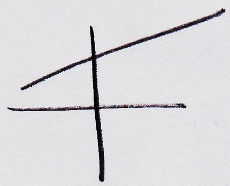
$y=\pm\sqrt{4-x^2}$ $\boxed{\text{No}}$

$x=0$ is mapped to both $\pm 2$

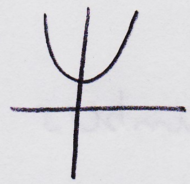
## Vertical Line Test

A set of points is the graph of a function if and only if every vertical line intersects the graph in at most 1 point.

**Ex:** Above

a) 

FUNCTION

b) 

FUNCTION

c) 

NOT A FUNCTION

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$x^2+y^2=4$ implicit function

$y=\pm\sqrt{4-x^2}$ explicit function (solve for y)

$x$ is the <u>independent</u> variable

$y$ " <u>dependent</u> "

## Restrictions on Domain:

1) Can't divide by zero
2) Can't take $\sqrt{x}$, $\sqrt[4]{x}$, $\sqrt[6]{x}$ etc. when $x < 0$

Ex: Find the domain

a) $y = x^2 + 1$

all real numbers

b) $y = \dfrac{x}{x^2 - 9}$

$x^2 - 9 \neq 0$

$(x-3)(x+3) \neq 0$

$x \neq \pm 3$

$\{x \mid x \neq \pm 3\}$

c) $y = \sqrt[3]{x}$

all real numbers

d) $y = \sqrt{x+2}$

$x + 2 \geq 0$

$x \geq -2$

$\{x \mid x \geq -2\}$

e) $y = \frac{1}{\sqrt{x+2}}$

$x+2 > 0$
$x > -2$

$\{x \mid x > -2\}$

f) $y = \frac{\sqrt{x+2}}{x-1}$

$x+2 \geq 0$ and $x-1 \neq 0$
$x \geq -2$ $\quad$ $x \neq 1$

$\{x \mid x \geq -2 \text{ and } x \neq 1\}$

Ex: Given $f(x) = x^2$. Find :

f is the mapping that takes $x \to x^2$

a) $f(2)$
$= 4$

b) $f(-2)$
$= (-2)^2 = 4$

c) $f(x) + f(2)$
$= x^2 + 4$

d) $2f(x)$
$= 2x^2$

e) $f(2x)$
$= (2x)^2$
$= 4x^2$

f) $f(-x)$
$= (-x)^2$
$= x^2$

g) $-f(x)$
$= -x^2$

h) $f(x+1)$
$= (x+1)^2$
$= x^2+2x+1$

i) $f(x+h)$
$= (x+h)^2$
$= x^2+2xh+h^2$

$$\boxed{\text{Difference Quotient of } f(x): \quad \frac{f(x+h)-f(x)}{h}}$$

Ex: Find the difference quotient

a) $f(x) = x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x+h$$

b) $f(x) = \sqrt{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{(\cancel{x}+h) - \cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

c) $f(x) = \frac{-2}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{-2}{x+h} + \frac{2}{x}\right]$$

$$= \frac{1}{h}\left[\frac{-2x + 2(x+h)}{(x+h)x}\right]$$

$$= \frac{1}{h}\left[\frac{2h}{(x+h)x}\right]$$

$$= \frac{2}{(x+h)x}$$

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| Function | Domain |
|---|---|
| $f+g$ | All $x$ in the domains of both $f$ and $g$ |
| $f-g$ | " |
| $f \cdot g$ | " |
| $\frac{f}{g}$ | All $x$ in the domains of both $f$ and $g$, and for which $g(x) \neq 0$ |

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Ex: $f(x) = x^3$ $g(x) = x^2 - 2$ Find:

a) $(f+g)(x) = f(x) + g(x) = x^3 + x^2 - 2$

b) $(f-g)(x) = f(x) - g(x) = x^3 - x^2 + 2$

c) $(f \cdot g)(x) = f(x)g(x) = x^3(x^2-2) = x^5 - 2x^3$

d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3}{x^2-2}$

Ex: Domain of $\sqrt{x} + \frac{1}{x+5}$ ?

Domain of $\sqrt{x}$ : $x \geq 0$

$\frac{1}{x+5}$ : $x \neq -5$

Domain of $\sqrt{x} + \frac{1}{x+5}$ : $\{x \mid x \geq 0 \text{ and } x \neq -5\}$