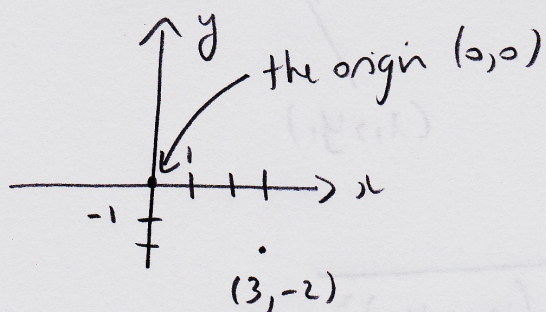


2.1 Distance and Midpoint

p.1

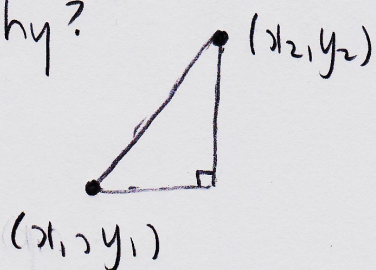


Q II $x < 0$ $y > 0$	Quadrant I $x > 0$ $y > 0$
Q III $x < 0$ $y < 0$	Q IV $x > 0$ $y < 0$

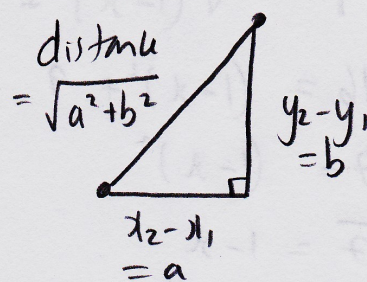
The distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Why?



→



Ex: Find distance between $P(-6, 2)$ and $Q(-4, 7)$

$\uparrow$
 (x_2, y_2)

$\nearrow$
 (x_1, y_1)

$$\begin{aligned}
 d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-4 + 6)^2 + (7 - 2)^2} \\
 &= \sqrt{2^2 + 5^2} \\
 &= \sqrt{29}
 \end{aligned}$$

Ex: Find all points on the x -axis that are 4 units from $(1, 3)$.

$\nearrow$
 (x_2, y_2)

$\uparrow$
 y
 $\rightarrow x$

Points on x -axis have $y=0$
 $(x_1, y_1) = (x, 0)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$4 = \sqrt{(1 - x)^2 + 3^2}$$

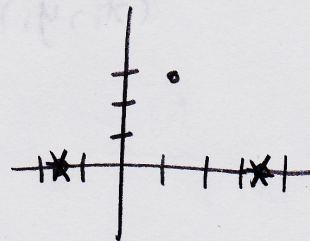
$$16 = (1 - x)^2 + 9$$

$$7 = (1 - x)^2$$

$$\pm\sqrt{7} = 1 - x$$

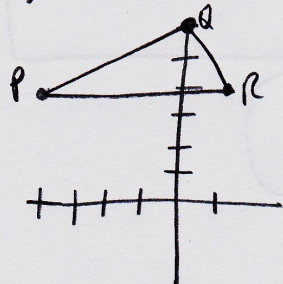
$$x = 1 \mp \sqrt{7}$$

Answer: $(1 + \sqrt{7}, 0)$ and $(1 - \sqrt{7}, 0)$



Ex: Form a triangle from $P(-4,4)$, $Q(0,6)$ and $R(1,4)$ p.3

a) Find each side length



$$d(P,Q) = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$$

$$d(P,R) = 5$$

$$d(Q,R) = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

b) Is the triangle a right triangle?

Check if $a^2 + b^2 = c^2$

longest side

$$(2\sqrt{5})^2 + (\sqrt{5})^2 \stackrel{?}{=} 5^2$$

$$4(5) + 5 = 25 \quad \checkmark$$

Yes

c) Find the area

both touching right angle

$$\text{Area} = \frac{1}{2} \text{base} \cdot \text{height}$$

$$= \frac{1}{2} d(P,Q) \cdot d(Q,R)$$

$$= \frac{1}{2} (2\sqrt{5}) \cdot \sqrt{5}$$

$$= 5$$

p. 4
The midpoint of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

average of
x-values

Ex: Find the midpoint of AB

$$A = (-3, 11) \quad B = (-7, 4)$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-10}{2}, \frac{15}{2} \right)$$

$$= \left(-5, \frac{15}{2} \right)$$

Ex: The midpoint of the line segment from $P_1 = (-8, 7)$ to P_2 is $(-6, 6)$. Find P_2 .

$$\text{Let } P_2 = (x_2, y_2)$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(-6, 6) = \left(\frac{-8 + x_2}{2}, \frac{7 + y_2}{2} \right)$$

$$-6 = \frac{-8 + x_2}{2}$$

$$-12 = -8 + x_2$$

$$-4 = x_2$$

$$6 = \frac{7 + y_2}{2}$$

$$12 = 7 + y_2$$

$$5 = y_2$$

$$\boxed{P_2 = (-4, 5)}$$