13.2 Arithmetic Sequences

A sequence is <u>arithmetic</u> if the difference between consecutive terms is constant.

d: Common difference

Ex: 7, 10, 13, 16, ... is arithmetic
(+3, +3, +3)

1st term $a_1 = 7$

d = Common difference
$= a_n - a_{n-1}$ (any term − previous term)
$= 3$

Ex: Is the sequence arithmetic? If so, find a_1 and d

a) $\{a_n\} = \{2^n\}$

2, 4, 8, 16, ...

Not arithmetic

b) $\{a_n\} = \{2 - 7n\}$

−5, −12, −19, ...

Arithmetic ✓

$a_1 = -5$

$d = a_n - a_{n-1}$
$= -12 - (-5)$
$= -7$

n^{th} term of an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

$a_1, a_2, a_3, \ldots, a_n$

$a_2 = a_1 + d$, $a_3 = a_1 + 2d$, $a_n = a_1 + (n-1)d$

Ex: Find the 30^{th} term of $3, 8, 13, 18, \ldots$

Arithmetic $a_1 = 3$ $d = 5$

$$a_n = a_1 + (n-1)d$$
$$a_{30} = a_1 + 29d$$
$$= 148$$

Ex: An arithmetic sequence has $a_5 = -3$ and $a_{10} = -13$. Find a formula for a_n.

→ Find a_1 and d

$a_5 = -3$

$a_1 + 4d = -3$ ①

$a_{10} = -13$

$a_1 + 9d = -13$ ②

② $a_1 + 9d = -13$

− ① $a_1 + 4d = -3$

$5d = -10$

$d = -2$

+ either equation

① $a_1 + 4(-2) = -3$

$a_1 = 5$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)(-2)$

or $a_n = 7 - 2n$

Sum of the first n terms of an arithmetic sequence

$S_n = \frac{n}{2}[a_1 + a_n]$

or $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Ex: Find the sum of the first 20 terms of $6, 7.5, 9, \ldots$

Arithmetic $a_1 = 6$ $d = 1.5$ $n = 20$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{20} = \frac{20}{2}[2(6) + 19(1.5)]$

$= 405$

Ex: Find the sum $2 + 5 + \ldots + 200$

Arithmetic $a_1 = 2$ $d = 3$ $a_n = 200$

$n = ?$

$a_n = 200$

$a_1 + (n-1)d = 200$

$2 + (n-1)(3) = 200$

$n - 1 = 66$

$n = 67$

$$S_n = \frac{n}{2}[a_1 + a_n]$$

$$S_{67} = \frac{67}{2}[2 + 200]$$

$$= 6767$$

Recursive definition of a sequence: define a_n in terms of previous terms

Recursive definition of an arithmetic sequence:

$$a_n = a_{n-1} + d \quad n \geq 2$$ and indicate a_1

Ex: Give a recursive definition of a) 3, 5, 7, ...

$$a_n = a_{n-1} + 2 \quad n \geq 2 \qquad a_1 = 3$$

b) $-7, -9, -11, \ldots$

$$a_n = a_{n-1} - 2 \quad n \geq 2 \qquad a_1 = -7$$