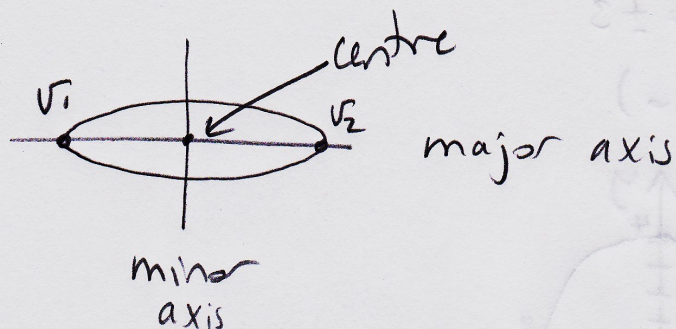
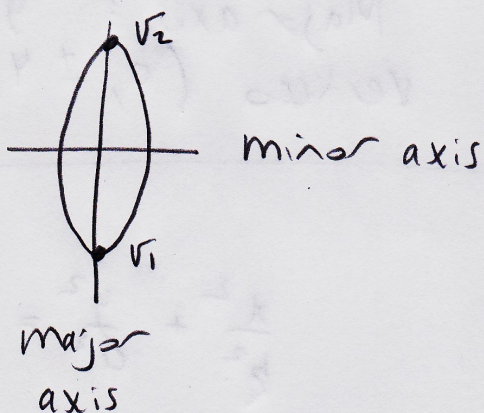


11.3 The Ellipse

p. 1



v_1, v_2 : vertices (on major axis)



Ex: Graph $\frac{x^2}{9} + \frac{y^2}{16} = 1$
Find centre and vertices

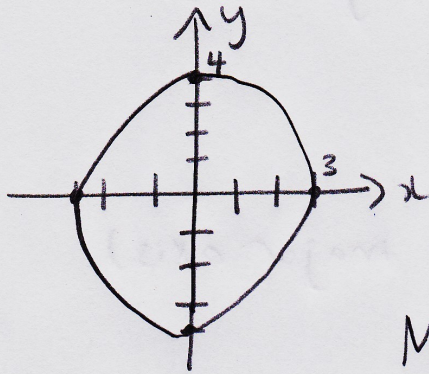
Plot 4 points : $x = 0 \Rightarrow \frac{y^2}{16} = 1$
 $y^2 = 16$
 $y = \pm 4$

$(0, 4), (0, -4)$

$$y=0 \Rightarrow \frac{x^2}{9} = 1$$

$$x = \pm 3$$

$$(3, 0), (-3, 0)$$



$$\text{Centre} = (0, 0)$$

Major axis : y-axis
vertices $(0, \pm 4)$

$$\text{Let } a > b > 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Centre} = (0, 0)$$

Major axis = x-axis

$$\text{Vertices} = (\pm a, 0)$$

$$\text{Other Points} = (0, \pm b)$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$(0, 0)$$

y-axis

$$(0, \pm a)$$

$$(\pm b, 0)$$

Shift ellipse h units right
 k units down

p.3

Let $a > b > 0$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Centre = (h, k)

(h, k)

Major axis is parallel
to x -axis

y -axis

Vertices $(h \pm a, k)$

$(h, k \pm a)$

Other Points $(h, k \pm b)$

$(h \pm b, k)$

Ex: Graph $\frac{(x-2)^2}{36} + \frac{(y+3)^2}{25} = 1$

$$\frac{(x-2)^2}{6^2} + \frac{(y+3)^2}{5^2} = 1$$

Centre = $(2, -3)$

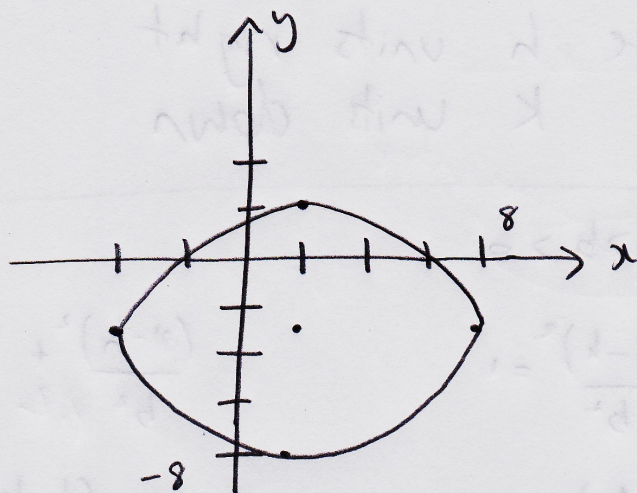
Major axis is parallel to x -axis

Vertices = $(h \pm a, k)$

= $(2 \pm 6, -3)$

= $(-4, -3)$ and $(8, -3)$

Other Points = $(h, k \pm b) = (2, -3 \pm 5) = (2, -8)$ and $(2, 2)$



Ex: Find the standard form

$$4x^2 + 3y^2 + 8x - 12y = -4$$

$$4(x^2 + 2x) + 3(y^2 - 4y) = -4$$

$$\downarrow$$

$$\left(\frac{2}{2}\right)^2 = 1$$

$$\downarrow$$

$$\left(\frac{-4}{2}\right)^2 = 4$$

Add 4(1)

Add 3(4)

$$4(x^2 + 2x + 1) + 3(y^2 - 4y + 4) = -4 + 4 + 12$$

$$4(x+1)^2 + 3(y-2)^2 = 12$$

$$\frac{(x+1)^2}{3} + \frac{(y-2)^2}{4} = 1$$

$$\frac{(x+1)^2}{\sqrt{3}^2} + \frac{(y-2)^2}{2^2} = 1$$