

1.4 Radical Equations

p.1

Ex: Solve $\sqrt[3]{4x+3} - 3 = 0$

Isolate root: $\sqrt[3]{4x+3} = 3$

Cube both sides: $4x+3 = 27$

$$4x = 24$$

$$x = 6$$

Check: $LS = \sqrt[3]{27} - 3 = 0$

$RS = 0$ ✓

{6}

Ex: Solve $\sqrt{x+4} + x = 8$

$$\sqrt{x+4} = 8-x$$

$$x+4 = (8-x)^2$$

$$x+4 = 64 - 16x + x^2$$

$$0 = 60 - 17x + x^2$$

$$(x-12)(x-5) = 0$$

$$x = 5, 12$$

Check $x=5$ ✓

$x=12$ ✗

$x=12$ is "extraneous"

{5}

Squaring both sides
can produce extraneous
solutions!

Ex: Solve $\sqrt{x} = \sqrt{x+16} - 2$

p. 2

Isolate more complicated root

$$\sqrt{x} + 2 = \sqrt{x+16}$$

$$(\sqrt{x} + 2)^2 = x + 16$$

$$x + 4\sqrt{x} + 4 = x + 16$$

$$4\sqrt{x} = 12$$

$$\sqrt{x} = 3$$

$$x = 9$$

Check $x=9$ ✓ $\{9\}$

Equations in Quadratic Form

Ex: Solve $(x^2-9)^2 + 6(x^2-9) + 8 = 0$

Let $y = x^2 - 9$:

$$y^2 + 6y + 8 = 0$$

$$(y+2)(y+4) = 0$$

$$(x^2-7)(x^2-5) = 0$$

$$\swarrow$$
$$x^2 - 7 = 0$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

$$\downarrow$$

$$x = \pm\sqrt{5}$$

$$\{ \pm\sqrt{7}, \pm\sqrt{5} \}$$

Ex: Solve $x + 2\sqrt{x} - 35 = 0$

Let $y = \sqrt{x}$ $(\sqrt{x})^2 + 2\sqrt{x} - 35 = 0$

$$y^2 + 2y - 35 = 0$$

$$(y + 7)(y - 5) = 0$$

$$(\sqrt{x} + 7)(\sqrt{x} - 5) = 0$$

$$\swarrow$$

$$\sqrt{x} + 7 = 0$$

$$\sqrt{x} = -7$$

no real solution

$$\downarrow$$

$$\sqrt{x} - 5 = 0$$

$$\sqrt{x} = 5$$

$$x = 25$$

Check $x = 25$ ✓

$\{25\}$

Ex:

Grouping

Ex: Solve $2x^3 - x^2 - 32x + 16 = 0$

$$x^2(2x - 1) - 16(2x - 1) = 0$$

$$(x^2 - 16)(2x - 1) = 0$$

$$\swarrow$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

$$\downarrow$$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

Check all ✓

$\{\pm 4, \frac{1}{2}\}$