# 1.2 Quadratic Equations

Solve $ax^2+bx+c=0 \quad (a \neq 0)$

## Method I: Factoring

Ex: Solve

a) $x^2+7x=0$

$x(x+7)=0$

$x=0 \qquad x+7=0$

$x=-7$

$\{0,-7\}$

b) $x^2+4x+4=0$

$(x+2)^2=0$

$x=-2$

$\{-2\}$

c) $2x^2-3=-5x$

$2x^2+5x-3=0 \qquad 6,-1$

$2x^2+6x-x-3=0$

$2x(x+3)-(x+3)=0$

$(2x-1)(x+3)=0$

$x=\frac{1}{2},-3 \qquad \left\{\frac{1}{2},-3\right\}$

# Method II: Square Root Method

Let $p > 0$

$x^2 = p$ has 2 solutions : $x = \pm\sqrt{p}$

Ex: Solve

a) $x^2 = 17$

$x = \pm\sqrt{17}$

b) $(x+7)^2 = 9$

$x + 7 = \pm\sqrt{9}$

$x = -7 \pm 3$

$x = -10, -4$

c) $x^2 = -16$

no solution

(disregard complex #)

Why? $x^2 = p$

$x^2 - p = 0$

$(x - \sqrt{p})(x + \sqrt{p}) = 0$

$x = \pm\sqrt{p}$

# Method III: Completing the Square

<u>Ex</u>: Solve by completing the square

a) $x^2+8x+6=0$

$$x^2+8x=-6$$

$$\left(\frac{8}{2}\right)^2=16$$

$$x^2+8x+16=10$$

$$(x+4)^2=10$$

$$x+4=\pm\sqrt{10}$$

$$x=-4\pm\sqrt{10}$$

b) $3x^2+36x+20=0$

Make leading coefficient $=1$

$$x^2+12x+\frac{20}{3}=0$$

$$x^2+12x=-\frac{20}{3}$$

$$\left(\frac{12}{2}\right)^2=36$$

$$x^2+12x+36=36-\frac{20}{3}$$

$$(x+6)^2=\frac{88}{3}$$

$$x+6=\pm\sqrt{\frac{88}{3}} \leftarrow \pm\frac{\sqrt{4}\sqrt{22}}{\sqrt{3}}=\pm\frac{2\sqrt{66}}{3}$$

$$\boxed{x=-6\pm\frac{2\sqrt{66}}{3}}$$

# Method IV: Quadratic Formula

$ax^2+bx+c=0$ has solutions

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex: Solve with the quadratic formula

a) $3x^2+7x+2=0$

$$x = \frac{-7 \pm \sqrt{25}}{6}$$

$$x = \frac{-7 \pm 5}{6}$$

$$x = -2, \frac{-1}{3}$$

b) $6x^2+3x+8=0$

$$x = \frac{-3 \pm \sqrt{-183}}{12}$$

no real solution

c) $x^2+7x+\frac{49}{4}=0$

$$4x^2+28x+49=0$$

$$x = \frac{-28 \pm \sqrt{0}}{8}$$

$$x = \frac{-28}{8} = -7/2$$

$b^2-4ac$ is called the <u>discriminant</u> of the quadratic equation

| | | |
|---|---|---|
| If $b^2-4ac > 0$ | then $ax^2+bx+c=0$ | has 2 different real solutions |
| $= 0$ | | 1 repeated solution |
| $< 0$ | | no real solution |

<u>Ex</u>: How many real solutions?

a) $x^2+18x+12=0$

$b^2-4ac = 276 > 0$

$\boxed{2}$

b) $x^2+18x+81=0$

$b^2-4ac = 0$

$\boxed{1}$

<u>Ex</u>: Solve $5 + \frac{2}{x} - \frac{7}{x^2} = 0$

$$5x^2 + 2x - 7 = 0$$

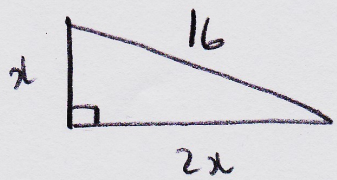
$$x = \frac{-2 \pm \sqrt{144}}{10}$$

$$x = \frac{-2 \pm 12}{10}$$

$$x = \frac{-14}{10}, \frac{10}{10}$$

$$x = -7/5, 1$$

<u>Ex</u>: A right triangle has a hypotenuse of length 16m. Find the lengths of the shorter sides if one is twice the length of the other.




$$x^2 + (2x)^2 = 16^2$$

$$x^2 + 4x^2 = 256$$

$$5x^2 = 256$$

$$x^2 = \frac{256}{5}$$

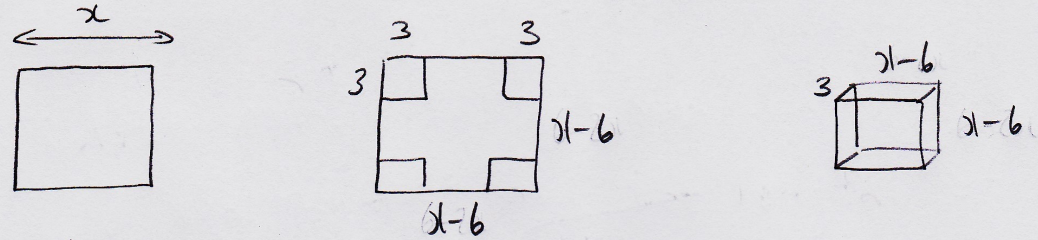
$$x = \pm\sqrt{\frac{256}{5}}$$

$$x = \pm \frac{16\sqrt{5}}{5}$$

Length > 0 therefore $x = \frac{16\sqrt{5}}{5}$

The side lengths are $\frac{16\sqrt{5}}{5}$ m and $\frac{32\sqrt{5}}{5}$ m.

**Ex:** Cut a square of side 3 cm out of a square piece of metal to produce an open-topped box with volume 192 $cm^3$. What is the original side length?




Want $x$

$$\text{Volume} = 192$$
$$3(x-6)^2 = 192$$
$$(x-6)^2 = 64$$
$$x-6 = \pm 8$$
$$x = 6 \pm 8$$
$$x = -2, 14$$

Length > 0 therefore $x = 14$ cm